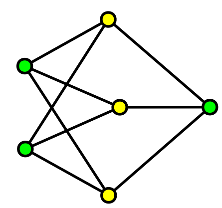
Friday's annotated slides

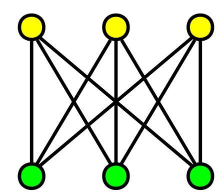
MA284 : Discrete Mathematics

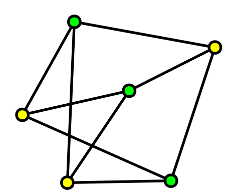
Week 8: The fundamentals of graph theory;
Planar Graphs
25 and 27 October, 2017

1. Definitions
   - ~~1. A graph~~
   - ~~2. Paths and connected graphs~~
   - ~~3. Complete graphs~~
   - ~~4. Vertex degree~~
   - 5. Bipartite graphs
   - 6. Subgraphs
   - 7. Named graphs
2. Planar graphs
   - Faces, edges and vertices
   - Euler's formula
3. ~~Non-planar graphs~~
4. Exercises

Today



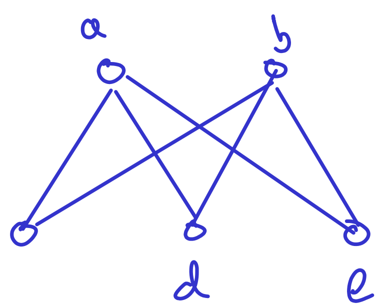




See also Sections 4.1 and 4.2 of Levin's *Discrete Mathematics: an open introduction*.

If it is possible to divide the vertex set, V, into two disjoint sets, V_1 and V_2, such that there are no edges between any two vertices in the same set, then the graph is **BIPARTITE**.

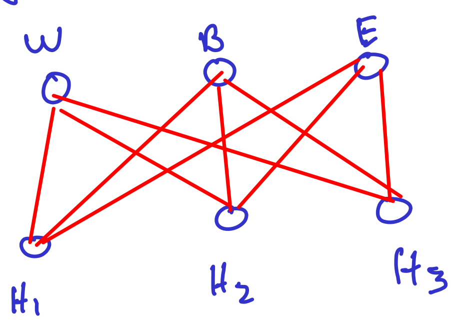
Eg



$V_1 = \{a, b\}$, $V_2 = \{c, d, e\}$

Finished here Wednesday

Eg (Utility Graph)

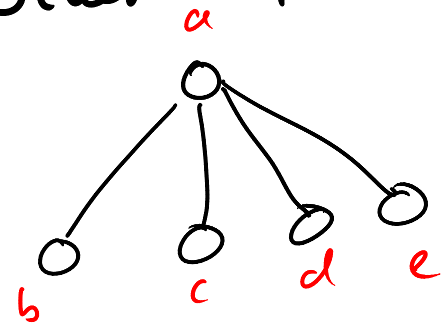


$V_1 = \{W, B, E\}$

$V_2 = \{H_1, H_2, H_3\}$

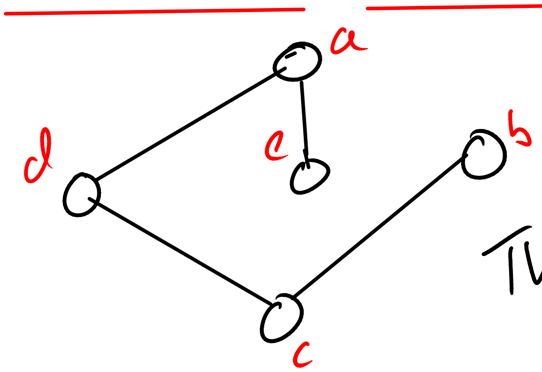
If it is possible to divide the vertex set, V, into two disjoint sets, V_1 and V_2, such that there are no edges between any two vertices in the same set, then the graph is **BIPARTITE**.

Other bipartite graphs



$V_1 = \{a\}$

$V_2 = \{b, c, d, e\}$

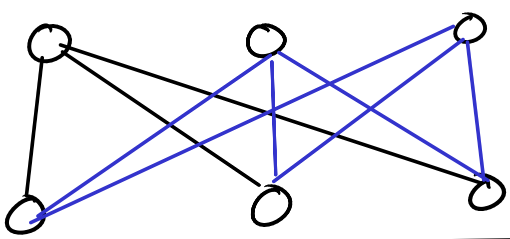


This is bipartite with, eg $V_1 = \{a, c\}$, $V_2 = \{b, d, e\}$.

There are other ways to partition.

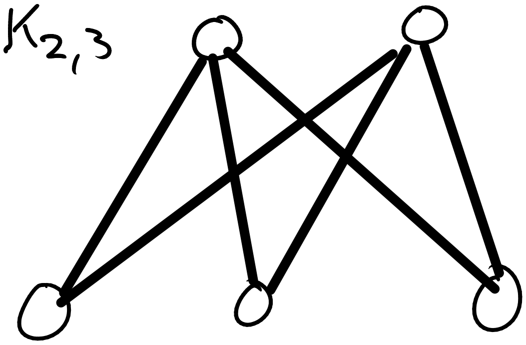
When the bipartite graph is such that *every* vertex in V_1 is connected to *every* vertex in V_2 (and *vice versa*) the graph is a **COMPLETE BIPARTITE GRAPH**. If $|V_1| = m$, and $|V_2| = n$, we denote it $K_{m,n}$.

Already we have seen:

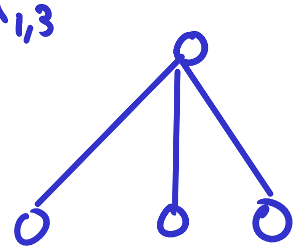


This is the utility graph: $K_{3,3}$.

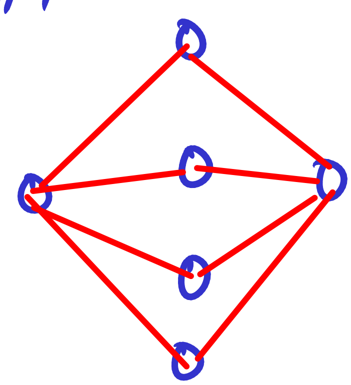
$K_{2,3}$



$K_{1,3}$



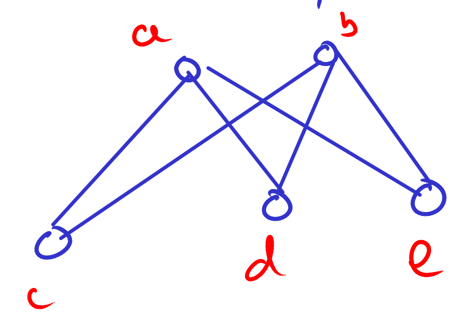
$K_{2,4}$



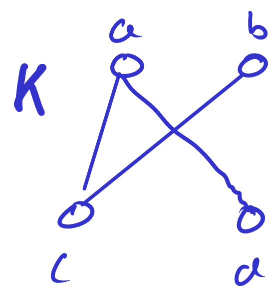
We say that $G_1 = (V_1, E_1)$ is a **SUBGRAPH** of $G_2 = (V_2, E_2)$ provided $V_1 \subset V_2$, and $E_1 \subset E_2$.

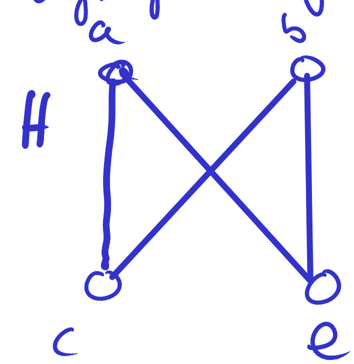
We say that $G_1(V_1, E_1)$ is an **INDUCED SUBGRAPH** of $G_2 = (V_2, E_2)$ provided that $V_1 \subset V_2$ and E_2 contains all edges of E_1 which are subsets of V_2 .

Eg $G_2 = K_{2,3}$



These are subgraphs of G_1





Here H is an induced subgraph of G_1

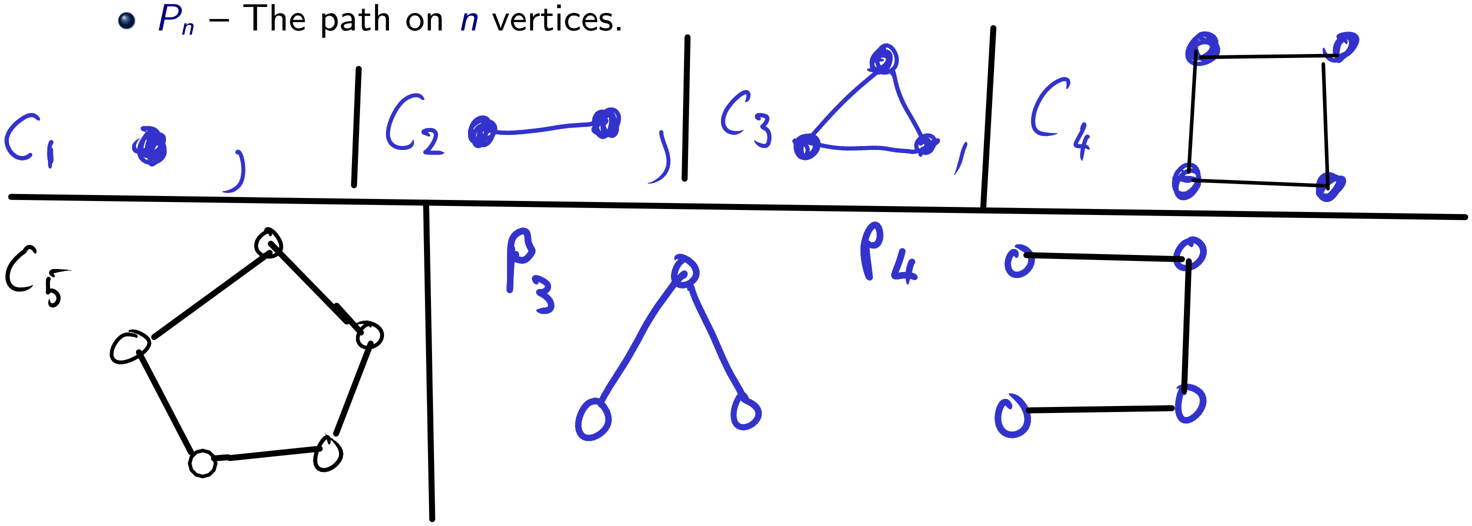
but K is not (the edge $\{b, d\}$ is "missing").

Some graphs are used more than others, and get special names. We already had

- K_n – the complete graph on n vertices.
- $K_{m,n}$ – The complete bipartite graph with sets of m and n vertices.

Other important ones include

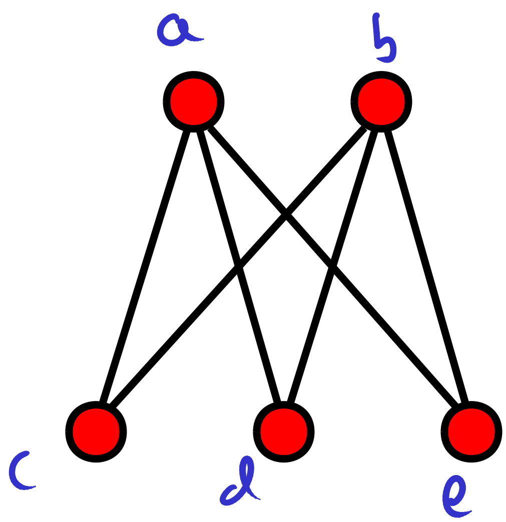
- C_n – The cycle on n vertices. ← "important".
- P_n – The path on n vertices.

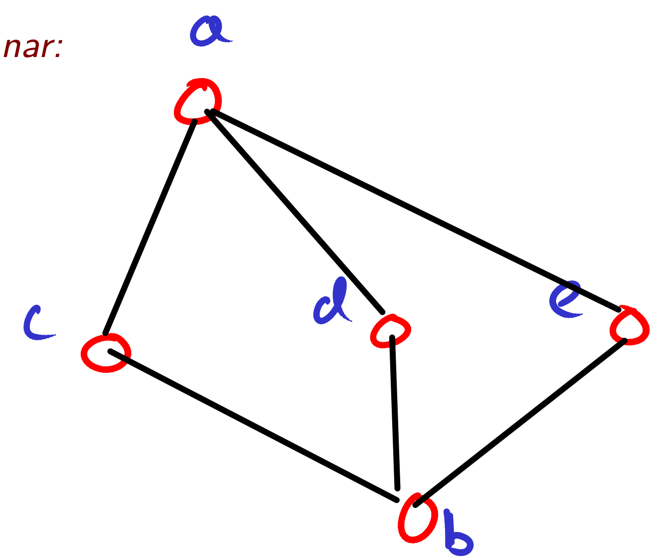


Planar graph

If you can sketch a graph so that none of its edges cross, then it is a *planar* graph.

Example: The Graph K_{23} is *planar:*

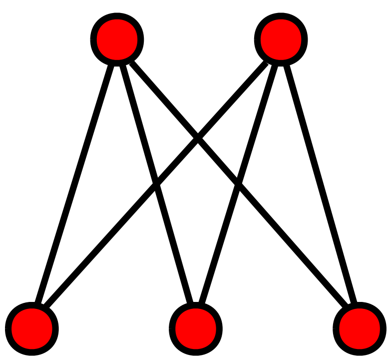


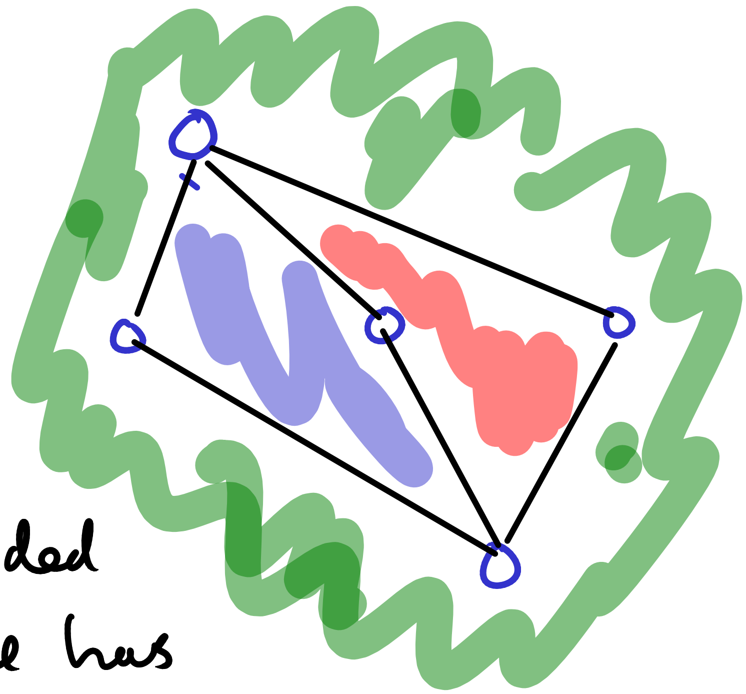


These graphs are *equal*. The sketch on the right (see annotated notes) is a *planar representation* of the graph.

When a planar graph is drawn without edges crossing, the edges and vertices of the graph divide the plane into regions. We will call each region a *face*.

Example: the planar representation of K_{23} has **3 faces** (because the "outside" region counts as a face).

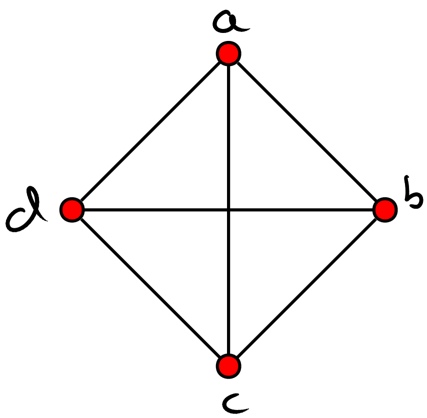




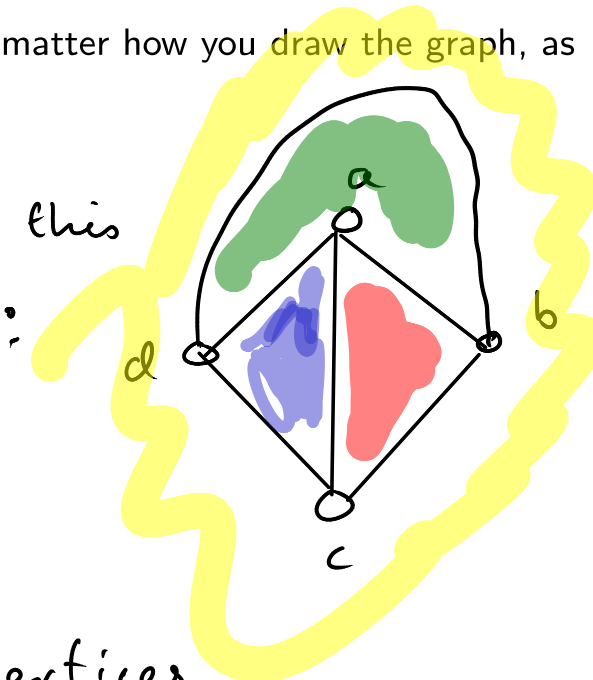
A face is a bounded region. Each face has at least 3 edges in its boundary.

The number of faces does not change no matter how you draw the graph, as long as no edges cross.

Example: Count the faces of K_4.



Draw this as:



So K_4 has $v = 4$ vertices
$e = 6$ edges.
and $f = 4$. faces