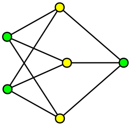
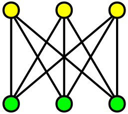
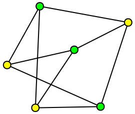
MA284 : Discrete Mathematics

Week 8: The fundamentals of graph theory; Planar Graphs 25 and 27 October, 2017







See also Sections 4.1 and 4.2 of Levin's *Discrete Mathematics: an open introduction*.

Assignment 2 is open

To access the assignment, go to
http://mathswork.nuigalway.ie/webwork2/1718-MA284

There are **20** questions. You may attempt each one up to **10** times.
This assignment contributes 10% to your final grade for Discrete Mathematics.

Deadline: 5pm, Friday, 27 October, 2017.

Thank you for your feedback on Discrete Mathematics. There were 56 responses. Here are a few:

2. The module is well organized

Agree	43
Agree somewhat	15
Unsure/not applicable	1
Disagree somewhat	0

3. I have access to sufficient materials to support my learning (for example: handouts, library books, Blackboard materials).

Agree	45
Agree somewhat	10
Unsure/not applicable	1
Disagree somewhat	3
Disagree	0

4. The feedback I have received is helping me improve my learning

Agree	19
Agree somewhat	15
Unsure/not applicable	17
Disagree	6
Disagree somewhat	2

5. The lectures are clear and well prepared.

Agree	48
Agree somewhat	9
Unsure/not applicable	2
Disagree somewhat	0
Disagree	0

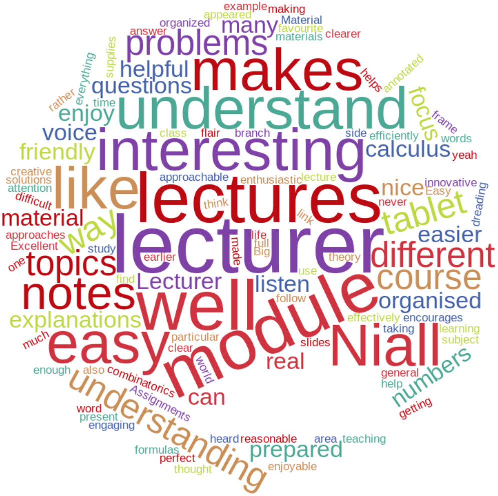
6. The lecturer(s) is/are helpful, approachable, and effective.

Agree	46
Agree somewhat	11
Disagree somewhat	2
Unsure/not applicable	0
Disagree	0

7. The tutorials/labs are helpful and effective.

Agree	29
Agree somewhat	10
Unsure/not applicable	15
Disagree somewhat	4
Disagree	1

10. What do you like about this module?



What suggestions can you offer that would help make this module a more valuable learning experience for you?

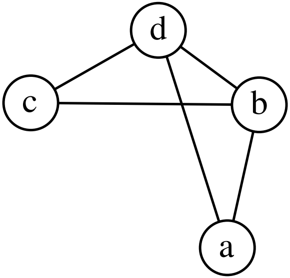
- More examples in lectures
- Module is too lecture based... students should feel free to shout questions out.
- I'd like to move faster but I know people might get left behind
- Less questions but more challanging ones would be great.
- More example of the type of questions that will appear on the exams
- More descriptive notes.
- Non-graded home work as well as the usual graded homework...
- Offer service similiar to clickers or something to make lecture more engaging
- The overhead lecture notes to be uploaded onto blackboard that are written during class,
- Shorter web works
- The tutorial should be improved

Recall from last week... a graph

A *GRAPH* is a collection of **vertices**, and **edges** joining pair of vertices.
A graph is defined in terms of its *edge set* and *vertex set*. That, the graph G with vertex set V and edge set E is written as

$$G = (V, E).$$

If two vertices are connected by an edge, we say they are *adjacent*.



A **PATH** is a sequence of adjacent vertices in a graph. We often will talk about a *path* between two vertices. And, since there can be more than one, the *SHORTEST PATH* is particularly important.

A graph is **CONNECTED** if there is a path between every pair of vertices.

A graph is **COMPLETE** if every pair of vertices are adjacent. This family of graphs is VERY important. They are denoted K_n – the complete graph on n vertices.

The **DEGREE** of a vertex is the number of edges emanating from it.

If we know the degree of every vertex in the graph then we know the number of edges.

Example (MA284 Semester 1 Exam, 2015/2016, Q3(a))

Among a group of five people,

(i) is it possible for everyone to be friends with exactly <u>two</u> of the other people in the group?

(ii) is it possible for everyone to be friends with exactly <u>three</u> of the other people in the group?

If it is possible to divide the vertex set, V, into two disjoint sets, V_1 and V_2, such that there are no edges between any two vertices in the same set, then the graph is **BIPARTITE**.

When the bipartite graph is such that *every* vertex in V_1 is connected to *every* vertex in V_2 (and *vice versa*) the graph is a **COMPLETE BIPARTITE GRAPH**. If $|V_1| = m$, and $|V_2| = n$, we denote it $K_{m,n}$.

We say that $G_1 = (V_1, E_1)$ is a **SUBGRAPH** of $G_2 = (V_2, E_2)$ provided $V_1 \subset V_2$, and $E_1 \subset E_2$.

We say that $G_1(V_1, E_1)$ is an **INDUCED SUBGRAPH** of $G_2 = (V_2, E_2)$ provided that $V_1 \subset V_2$ and E_2 contains all edges of E_1 which are subsets of V_2 .

Some graphs are used more than others, and get special names. We already had

- K_n – the complete graph on n vertices.
- $K_{m,n}$ – The complete bipartite graph with sets of m and n vertices.

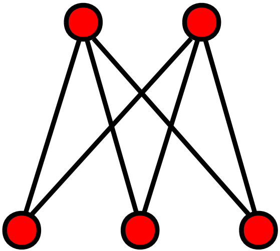
Other important ones include

- C_n – The cycle on n vertices.
- P_n – The path on n vertices.

Planar graph

If you can sketch a graph so that none of its edges cross, then it is a *planar* graph.

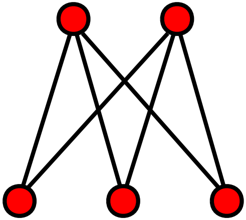
Example: The Graph K_{23} is *planar:*



These graphs are *equal*. The sketch on the right (see annotated notes) is a *planar representation* of the graph.

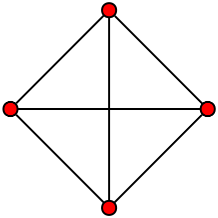
When a planar graph is drawn without edges crossing, the edges and vertices of the graph divide the plane into regions. We will call each region a *face*.

Example: the planar representation of K_{23} has **3 faces** (because the "outside" region counts as a face).



The number of faces does not change no matter how you draw the graph, as long as no edges cross.

Example: Count the faces of K_4.



More examples: Count the number of edges, faces and vertices in the cycle graphs C_3, C_4 and C_5.

More examples: Count the number of edges, faces and vertices in these planar graphs:

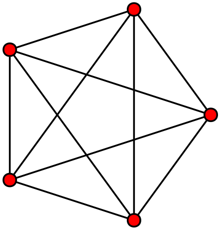
Euler's formula for planar graphs

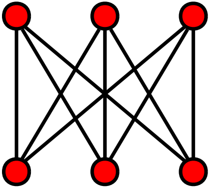
For any (connected) planar graph with v vertices, e edges and f faces, we have

$$v - e + f = 2$$

Outline of proof:

If course, most graphs do **not** have a planar representation. We have already met two that (we think) cannot be drawn so no edges cross: K_5 and K_{33}:

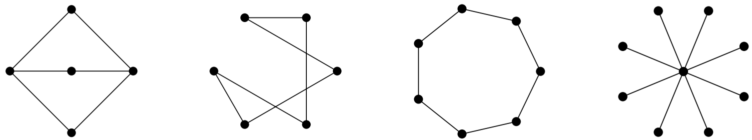




However, it takes a little work to *prove* that these are non-planar.

These questions are based on exercises in Section 4.1 of Levin's *Discrete Mathematics*. Solutions are also available from that book.

Q1. Which of the graphs below are bipartite? Justify your answers.



Q2. For which $n \geq 3$ is the graph C_n bipartite?

Q3. For each of the following, try to give two different unlabeled graphs with the given properties, or explain why doing so is impossible.

(a) Two different trees with the same number of vertices and the same number of edges. A tree is a connected graph with no cycles.
(b) Two different graphs with 8 vertices all of degree 2.
(c) Two different graphs with 5 vertices all of degree 4.
(d) Two different graphs with 5 vertices all of degree 3.