§0. Introduction to MA378

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MA378/531 – Numerical Analysis II ("NA2")

January 2017

This is a one-semester upper-level module on numerical analysis, taken by various cohorts, including Mathematics & Education, Mathematical Science, Mathematics, and Applied Mathematics.

The basic information for the course is as follows:

Lecturer: Dr Niall Madden, School Mathematics, Statistics and Applied Mathematics.
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Lectures: Monday at **5pm** in AC215, and Wednesday at 1pm in AC213.

Labs/tutorials: To be arranged.

Assessment

- Three MATLAB labs, each worth 2%.
- Two written assignments, each worth 8%.
- An in-class test, worth 8%.
- A 2-hour exam at the end of the semester, worth 70%.

The labs provide an opertunity for you to implement the algorithms we study, as well as their extensions and limitations.

The written assignments promote in-depth engagement with specific topics, while the class test encourages one to take a broad view of the module.

Text books

Main text: **Süli and Mayers, *An Introduction to Numerical Analysis***. Available from the library at 519.4 MAY, and the University book-shop.

Other useful books include

- G.W. Stewart, *Afternotes on Numerical Analysis*. *The full text is freely available online to NUI Galway users!* Not as formal as Süli and Mayers.
  epubs.siam.org/doi/book/10.1137/1.9781611971491
- Cleve Moler, *Numerical Computing with MATLAB*. The emphasis is on the implementation of algorithms in MATLAB. Also, it is freely available online.
- James F Epperson, *An introduction to numerical methods and analysis*. There are five copies in the library at 519.4.

Web site:

The on-line content for the course will be hosted at NUIGalway.BlackBoard.com and at `http://www.maths.nuigalway.ie/~niall/MA378`. There you'll find various pieces of information, including these notes/slides, problem sets, announcements, etc.

If you are registered for MA378, you should be automatically enrolled onto the blackboard site. If you are enrolled in MA531, please send an email to me.

The on-line notes are a synopsis of the course material. They are arranged section-by-section. They contain most of the main ideas, statements of theorems, results and exercises. They don't contain proofs of theorems, examples, solutions to exercises, etc. Please let me know of the typos and mistakes that you spot.

Each section of the notes has a set of exercises. *The homework assignments, class test, and final exam will be primarily based on these exercises*.

You should try to bring these notes to class, or be able to access them in class.

Slides used during class are based on the notes. They are will be on the website too, with an annotated version posted after class. There should not be any reason to print these.

Numerical analysis is the

- design
- analysis
- and implementation

of numerical algorithms that yield *exact* or *approximate* solutions to mathematical problems. The specific problems we will study are

1. Interpolation I: **Polynomial interpolation**.
2. Interpolation II: ***Piecewise polynomial "splines"***.
3. Numerical Integration I: **Newton-Cotes Quadrature**.
4. Numerical Integration II: ***Gaussian Quadrature***.
5. Numerical solution of Boundary Value Problems by the Finite Element Method.

Although these might seem like diverse topics, in each case we will attempt to find the most suitable polynomial that solves the problems.

The big idea is...

Suppose we have a problem to solve, for which we know there is a solution, but that the solution is very hard to find. Or maybe impossible to find. We replace the problem with one that is easier, but has a similar solution, and solve that instead.

While there are many variations, there is a single core idea we will return to again and again.

If the difficult problem is expressed in terms of some complicated function, then we approximate that function with a simple polynomial, usually of degree 3 or less. Choosing that polynomial is the "**design part**", and is usually quite interesting.

We then have to devise a set of steps for computing that polynomial, and finding the solution to our particular problem. This is the "**implementation**" stage.

When this is done by hand, it can easily become very boring. But through the use of computers, it take on an important, creative role in the process.

Finally we have the "**analysis**" part: this is the most interesting and mathematically challenging aspect: *can we say how close our approximate solution is to true solution?*

That we can answer this question in a precise manner is a bit surprising. For how can I give an accurate estimate for how close my approximation is to the true solution, when I don't know what that true solution is?

Anyone who can remember their first and second years of analysis and algebra should be able to handle this course. Students who know a little about differential equations (initial value and boundary value) will find a certain sections (particularly in Semester II) somewhat easier than those who haven't.

If its been a while since you covered basic calculus, you will find it very helpful to revise the following:

- the Intermediate Value Theorem;
- **Rolle's Theorem** and the The Mean Value Theorem;
- Taylor's Theorem,
- and the triangle inequality: $|a+b| \leq |a| + |b|$.

You'll find them in any good text-book, e.g., Appendix 1 of Süli and Mayers.

Lecture and Lab times

	Mon	Tue	Wed	Thu	Fri
9 – 10		❐			
10 – 11		❐			
11 – 12					
12 – 1					
1 – 2			✗		
2 – 3					
3 – 4					
4 – 5					
5 – 6	✗				