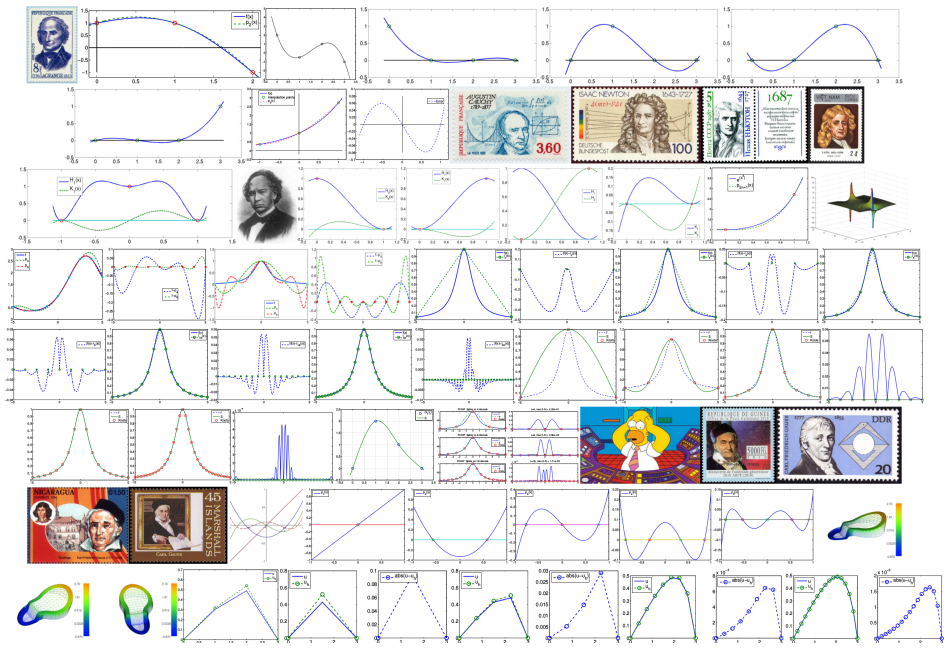
§4.5: Review of NA2

MA378/531 – Numerical Analysis II ("NA2")

March 2017



This module has 4 sections:

1. Polynomial Interpolation
2. Piecewise polynomial interpolation
3. Numerical Integration (Quadrature)
4. Finite element methods

Also: MATLAB

We now review — primarily to provide you with a check-list of topics.

1.1 The polynomial interpolation problem (PIP) in 2 forms;

1.2 Uniqueness (*if* $p_n \in \mathcal{P}_n$ *has* $n+1$ *zeros, then* $p_n \equiv 0$)

1.3 The Vandermonde matrix method (and its ill-conditioning)

1.4 **Lagrange Polynomials and Lagrange Interpolation**

1.5 Rolle's Theorem

1.6 **Cauchy's theorem**: $f(x) - p_n(x) = \frac{f^{(n+1)}(\tau)}{(n+1)!}\pi_{n+1}(x)$.

1.7 Synthetic Division (and the Newton Form of the Interpolant).

1.8 Hermite Interpolation: construction and error estimates.

1.9 Convergence & Runge's Example

2.1 Linear Interpolating Splines:

- construction,
- error analysis,
- best approximation, and
- the Minimum Energy property.

2.2 (Natural) Cubic Splines: construction (error analysis stated without proof)

2.3 PCHIP: Piecewise Cubic Hermite Interpolating Polynomial.

- construction,
- error analysis.

3.1 Newton-Cotes methods, and their relationship to Lagrange interpolation.

3.2 Undetermined Coefficients

3.3 The Trapezium rule, and (sharp) error estimates

3.4 Simpson's Rule and (non-sharp) error estimates

3.5 Precision

3.6 Composite rules

3.7 Gaussian Quadrature (and *Undetermined Coefficients* again)

3.8 A sequence of Orthogonal Monic Polynomials (and vector spaces and inner products); properties of orthogonal monic polynomials; their roots as quadrature points.

3.9 Precision and convergence of Gaussian Quadrature methods.

4.1 Boundary value problems, and Maximum Principles.

4.2 The **variational** (weak) **formulation** of the problem, and the uniqueness of its solution.

4.3 The **FEM**, including finite and infinite dimensional spaces.

4.4 **FE implementation**, including Galerkin basis functions ("hat functions"), how to construct the linear system, and why the system matrix is tridiagonal and symmetric.

4.5 Analysis, **Cea's Lemma**.

MA378 is assessed by

- Labs
- Assignments
- Written two-hour exam.

The exam will have **4 questions**: one on each of the sections of the module. Answer any 3. All have the same marks.

Thanks

Thanks for taking this module. I hope you have found it interesting, and that some of the topics covered are useful in your future mathematical efforts.