Chapter 0

MA378: Introduction

0.1 Welcome!

This is a one-semester upper-level module on numerical analysis, taken by various groups of students, including those in Mathematics & Education, Mathematical Science, Mathematics, and Applied Mathematics.

The basic information for the course is as follows:

Lecturer: Dr Niall Madden, School Mathematics, Statistics and Applied Mathematics.
Office: AdB-1013, Arás de Brún.
Email: Niall.Madden@NUIGalway.ie
Phone (091 49) 3803.

Lectures: Monday at 17.00 in AC215, and Wednesday at 13.00 in AC213.

Labs/tutorials: To be arranged.

Assessment:
- Three MATLAB labs, each worth 2%.
- Two written assignments, each worth 8%.
- An in-class test, worth 8%.
- A 2-hour exam at the end of the semester, worth 70%.

The labs provide an opertunity for you to implement the algorithms we study, as well as their extensions and limitations. The written assignments promote in-depth engagement with specific topics, while the class test encourages one to take a broad view of the module.

0.1.1 Books

Süli and Mayers [SM03] *An Introduction to Numerical Analysis*, Cambridge University Press. Library call number is 519.4 MAY.

This is the main text for the course. All topics in the semester are covered in the book, except for one very short section on numerical differentiation.

GW Stewart [S96] *Afternotes on Numerical Analysis*, (519.4 STE) serves as a good companion text. It covers much of the earlier parts of the course. The 2nd volume in the series [S98] *Afternotes goes to Graduate School* includes a section on Cubic Splines. (Differential equations are not covered).

These books are freely available online through the NUI Galway library, because we have a subscription to SIAM ebooks.

Cleve Moler [M04] *Numerical Computing with MATLAB*. The emphasis is on the implementation of algorithms in MATLAB, but the techniques are well explained and there are some nice exercises. Also, it is freely available online.

0.1.2 Website

The course web-site is hosted on BlackBoard. There you'll find various pieces of information, including these notes. Much of the material can also be accessed directly at `http://www.maths.nuigalway.ie/~niall/MA378`. All grades will be communicated through Backboard. For lab assignments, you will have to submit you work through Blackboard.

The **lecture notes** are organised section-by-section. The content relevant to a particular class should be available 24 hours before the class. The notes contain most of the main remarks, statements of theorems, results and exercises. However, they will not contain proofs of theorems, examples, solutions to exercises, etc. Please let me know of the typos and mistakes that you spot. Each section of the notes has a set of exercises. *The homework assignments, class test, and final exam will be primarily based on these exercises*.

If possible, *please print out the day's notes and bring them with you to class* (or be able to access them during the class). Slides used during class are based on the notes. They are will be on the website too.

0.1.3 Topics

Numerical analysis is the design, analysis and implementation of numerical algorithms that yield *exact* or *approx-*

imate solutions to mathematical problems. The specific problems we will study are

1. Interpolation I: Polynomial interpolation.
2. Interpolation II: Piecewise polynomial "splines".
3. Numerical Integration I: Newton-Cotes quadrature.
4. Numerical Integration II: Gaussian quadrature.
5. Numerical solution of Boundary Value Problems by the Finite Element Method.

Although these might seem like diverse topics, in each case we will attempt to find the most suitable polynomial that solves the problems.

0.1.4 What is NA2 really about?

The big idea is:

> *Suppose we have a problem to solve, for which we know there is a solution, but that the solution is very hard to find. Or maybe impossible to find. We replace the problem with one that is easier, but has a similar solution, and solve that instead.*

While there are many variations, there is a single core idea we will return to again and again. If the difficult problem is expressed in terms of some complicated function, then we approximate that function with a simple polynomial, usually of degree 3 or less. Choosing that polynomial is the "design part", and is usually quite interesting.

We then have to devise a set of steps for computing that polynomial, and finding the solution to our particular problem. This is the "implementation" stage. When this is done by hand, it can easily become very boring. But through the use of computers, it take on an important, creative role in the process.

Finally we have the "analysis" part: this is the most interesting and mathematically challenging aspect: *can we say how close our approximate solution is to true solution?* That we can answer this question in a precise manner is a bit surprising. For how can I give an accurate estimate for how close my approximation is to the true solution, when I don't know what that true solution is?

0.1.5 Mathematical Preliminaries

Anyone who can remember their first and second years of analysis and algebra should be able to handle this course. Students who know a little about boundary-value differential equations will find a certain sections a little easier than those who haven't.

If it is a while since you covered basic calculus, you will find it very helpful to revise the following: the Intermediate Value Theorem; **Rolle's Theorem**; The Mean Value Theorem; Taylor's Theorem, and the triangle inequality: $|a+b| \leqslant |a|+|b|$. You'll find them in any good text book, e.g., Appendix 1 of Süli and Mayers.

0.1.6 Licensing

Most of the images and illustrations have been created by me. The rest —mainly photos—are, to the best of my knowledge, in the public domain, and sources have been provided.

Most of the exercises, and some examples, are taken from the text-books listed below. Where this has been done, references are provided.

Please email Niall.Madden@NUIGalway if you think there is an attribution error.

Bibliography

[SM03] Endre Süli and David Mayers, *An Introduction to Numerical Analysis*, Cambridge University Press, 2003. 519.4 MAY.

[S96] G.W. Stewart, *Afternotes on Numerical Analysis*, SIAM, 1996. 519.4 STE. `http://epubs.siam.org/doi/book/10.1137/1.9781611971491`

[S98] G.W. Stewart, *Afternotes goes to Graduate School*, SIAM, 1998. 519.4 STE. `http://epubs.siam.org/doi/book/10.1137/1.9781611971422`

[SB92] Stoer and Bulirsch, *An Introduction to Numerical Analysis*, Springer.

[QSS00] Quarteroni, Sacco and Saleri, *Numerical Mathematics*, Springer.

[M04] Cleve Moler, *Numerical Computing with MATLAB*, Cambridge University Press. Also available free from http://www.mathworks.com/moler

[E02] James F Epperson, *An introduction to numerical methods and analysis*. `519.4EPP`