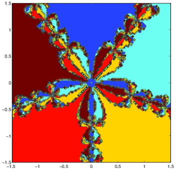
§0. Introduction to MA385; Taylor's Theorem

§0.1 Introduction

MA385/530 – Numerical Analysis 1

September 2017



This is a semester 1, upper level module on ***numerical analysis***. You may be taking this course if you are studying Mathematics, Applied Mathematics, Mathematical Science or Computer Science.

The basic information for the course is as follows:

Lecturer: Dr Niall Madden, School of Maths. My office is in room ADB-1013, Arás de Brún
Email: Niall.Madden@NUIGalway.ie

Lectures: 9am **Monday** and 3pm **Thursday** in AC201.

Tutorial/Lab: To begin during Week 3. Default time is Wednesday 6-5 (???).

Assessment:

- Written Homework Assignment 1 (10%)
- In-class test in Week 6 [tbc] (10%)
- Written Homework Assignment 2 (10%)
- 3 Computer labs (10%)
- A 2-hour exam in December (60%).

Main text: **Süli and Mayers, *An Introduction to Numerical Analysis***. Available from the library at 519.4 MAY, and the college book-shop. The scope of the book is almost perfect for the course, especially for those students taking both semesters. ***You should buy this book.*** Other useful books include

- G.W. Stewart, ***Afternotes on Numerical Analysis***. *The full text is freely available online to NUI Galway users!* Not as formal as Süli and Mayers.
- Cleve Moler, ***Numerical Computing with MATLAB***. The emphasis is on the implementation of algorithms in Matlab, but the techniques are well explained and there are some nice exercises. Also, it is freely available online.
- James F Epperson, ***An Introduction to Numerical Methods and Analysis***. There are five copies in the library at 519.4.
- Stoer and Bulirsch,*Introduction to Numerical Analysis*.

The on-line content for the course will be hosted at NUIGalway.BlackBoard.com and at http://www.maths.nuigalway.ie/MA385. There you'll find various pieces of information, including these notes/slides, problem sets, announcements, etc.

If you are registered for MA385, you should be automatically enrolled onto the blackboard site. If you are enrolled in MA530, please send an email to me.

These notes are a synopsis of the course material. My aim is to provide these in 4 sections, and always in advance of the class. They contain most of the main remarks, statements of theorems, results and exercises. However, they will not contain proofs of theorems, examples, solutions to exercises, etc.

You should try to bring these notes to class.
It will make it easier to follow the lecture.

Numerical analysis is the

(a) design,
(b) analysis and
(c) implementation of

numerical methods that yield ***exact or approximate*** solutions to mathematical problems.

It does not involve long, tedious calculations. We won't (usually) implement Newton's Method by hand, or manually do the arithmetic of Gaussian Elimination, etc.

(a) *Design* of a numerical method is perhaps the most interesting; its often about finding a clever way swapping the problem for one that is easier to solve, but has the same or similar solution. If the two problems have the same solution, then the method is ***exact***. If they are similar (but not the same), then it is ***approximate***.

The *Analysis* is the mathematical part; its usually culminates in proving a theorem that tells us (at least) one of the following.

- ***The method will work***: that our algorithm will yield the solution we are looking for;
- ***How much effort is required***
- If the method is approximate, determine ***how close the true solution be to the real one***. A description of this aspect of the course, to quote Epperson, is being "*rigorously imprecise* or *approximately precise*".

We'll look at the implementation of the methods in labs.

Topics

0. We'll start with a review of Taylor's theorem. It is central to the algorithms of the following sections.
1. Root-finding and solving non-linear equations.
2. Initial value ordinary differential equations.
3. Matrix Algorithms: solving systems of linear equations, and estimating eigenvalues and eigenvectors.

We also see how these methods can be applied to real-world, including Financial Mathematics.

When you have successfully completed this course, you will be able to demonstrate your factual knowledge of the core topics

- root-finding,
- solving ODEs,
- solving linear systems of equations and estimating eigenvalues,

using appropriate mathematical syntax and terminology.

Moreover, you will be able to describe the fundamental principles of the concepts (e.g., Taylor's Theorem) underpinning Numerical Analysis. Then, you will apply these principles to design algorithms for solving mathematical problems, and discover the properties of these algorithms.

Students will gain the ability to use a Matlab to implement these algorithms, and adapt the codes for more general problems, and for new techniques.

Anyone who can remember their first and second years of analysis and algebra should be well-prepared for this module. Students who know a little about differential equations (initial value and boundary value) will find a certain sections (particularly in Semester II) somewhat easier than those who haven't.

If its been a while since you covered basic calculus, you will find it very helpful to revise the following:

- the Intermediate Value Theorem;
- Rolle's Theorem and the The Mean Value Theorem;
- **Taylor's Theorem**,
- and the triangle inequality: $|a + b| \leq |a| + |b|$.

You'll find them in any good text-book, e.g., Appendix 1 of Süli and Mayers.

You'll also find it helpful to recall some basic linear algebra, particularly relating to eigenvalues and eigenvectors. Consider the statement:

> *"all the eigenvalues of a real symmetric matrix are real".*

If are unsure what the meaning of any of the terms used, or if you didn't know that its true, you should have a look at a book on Linear Algebra.

Why take this course?

Many industry and academic environments require graduates who can solve real-world problems using a mathematical model, but these models can often only be resolved using numerical methods. To quote one Financial Engineer: “We prefer approximate (numerical) solutions to exact models rather than exact solutions to simplified models”.

Another expert, who leads a group in fund management with DB London, when asked “what sort of graduates would you hire”, the list of specific skills included

- A programming language and a 4th-generation language such as MATLAB (or S-PLUS).
- Numerical Analysis.