MA385: Class Test, 19 October 2017

Answer all questions. Write your solutions, neatly, in the answer book provided.

. . . .

Q1. For each of the three following functions, show that it satisfies a Lipschitz condition, with respect to y, on the corresponding domain, and give an upper-bound for L:

(i) $f(t, y) = -y \quad \text{for} \quad t > 0,$ [10 Marks]

(ii) $f(t, y) = 3y/\sqrt{t} \quad \text{for} \quad t \in [1, \infty),$ [10 Marks]

(iii) $f(t, y) = 2 + t\cos(y) \quad \text{for} \quad 1 \leq t \leq 2.$ [20 Marks]

Q2. State Euler's method for computing an approximate solution to the initial value problem (IVP)

$$y(t_0) = y_0, \qquad y'(t) = f(t, y) \text{ for } t > t_0, \tag{1}$$

and show how it can be derived from a truncated Taylor series expansion. [20 Marks]

Q3. We learned in class that the global error of Euler's method satisfies

$$|\mathcal{E}_n| \leq \frac{h}{2} \max_{t_0 \leq t \leq t_1} |y''(t)| \frac{1}{L}(e^{L(t_n - t_0)} - 1). \tag{2}$$

Consider the IVP:

$$y(0) = 2, \qquad y'(t) = -y(t).$$

Determine from the smallest value of n should we take to ensure the error at $t = 1$ is no more that 10^{-3}? (Note that you already calculated L in Q1–(i) above). [40 Marks]