MA385: Sample Test, October 2017

This is a selection of exercises to help you prepare for the class test which will take place on Thursday, 19 October. The questions on the test will be different, but of a similar standard and length.

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An *Initial Value Problem (IVP)* is a differential equation of the form:

$$y(t_0) = y_0, \qquad y'(t) = f(t, y) \text{ for } t > t_0, \tag{1}$$

1. (a) State Euler's method for computing an approximate solution to an IVP.

   (b) Show how to motivate it with a Taylor series expansion, and derive an expression for the truncation error at step:

$$T_i := \frac{y(t_{i+1}) - y(t_i)}{h} - \Phi(t_i, y(t_i); h).$$

2. Suppose we want an approximation for the solution to the IVP

$$y(1) = 2, \qquad y'(t) = \frac{1}{1+y} \text{ for } t > 1. \tag{2}$$

   at the point $t = 2$.

   (a) Show that $y(t) \geq 2$ for all $t \geq 1$.

   (b) Show that (2) has a solution. **That is, show that $f(t, y) = 1/(1 + y)$ satisfies a *Lipschitz condition* with respect to y on for all $y \geq 1$.** Give an estimate for the Lipschitz constant, L.

   (c) We know that the global error, $\mathcal{E}_n = y(t_n) - t_n$, satisfies

$$|\mathcal{E}_n| \leq \frac{T}{L}(e^{L(t_n - t_0)} - 1), \tag{3}$$

   where $T = \max_{i=0,1,\ldots n} |T_i|$. Use this to give an upper bound for the error for Euler's method when $n = 10$ and $t_n = 2$.