

MA211 – Problem Set 5

Q19.1 Evaluate each of the following improper integrals:

(i) $\int_1^{\infty} \frac{1}{\ln(e^x)} dx.$ (ii) $\int_0^{\infty} \frac{x^2}{1+x^2} dx.$
 (iii) $\int_3^{\infty} \frac{dx}{(2x-1)^{2/3}}$ (iv) $\clubsuit \int_0^{\infty} \frac{x}{1+2x^2} dx$
 (v) $\int_0^{\infty} \frac{1}{1+e^x} dx$

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Q20.1 For each of the following integrals, determine if they converge or diverge

(i) $\int_1^{\infty} \frac{|\cos(x)|}{x^3+2} dx.$ (ii) $\int_0^1 \frac{dx}{x^{5/3}}$
 (iii) $\int_0^1 \frac{dx}{x^{3/5}}$ (iv) $\int_0^{\infty} \frac{x}{x^{3/2}+2x^2} dx.$
 (v) $\int_{-2}^2 \frac{1}{x^2} dx$ (vi) $\int_1^{\infty} \frac{1}{\sqrt{x+x^4}} dx$

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Q21.1 Find the general solutions to the following differential equations:

(i) $\frac{dy}{dx} = \frac{x}{y}$ (ii) $\frac{dy}{dx} = \frac{y}{x}$
 (iii) $\frac{dy}{dx} = y \ln(x)$ (iv) $\frac{dy}{dx} = \frac{y}{2x}.$
 (v) $\frac{dy}{dx} = \frac{e^x}{\sin(y)}$ (vi) $\frac{dy}{dx} = e^y \sin(x).$
 (vii) $\clubsuit \frac{dy}{dx} = \frac{\ln(x)}{xy^2}$

Q21.2 Solve the following initial value problems:

(i) $\frac{dy}{dx} = 3 + e^y; \quad y(0) = 1.$
 (ii) $\clubsuit \frac{dy}{dx} = \sinh(x)e^{-y} \quad y(0) = 1;$

Q21.3 For each of the following functions, determine if it is homogeneous. If it is homogeneous, then to what degree?

(i) $f = \sqrt{x^2 + y^2}$
 (ii) $f = \frac{x^2 + xy}{xy + y^2}$
 (iii) $f = xy^2$
 (iv) $f = \frac{2xy}{x^2 + y^2}$
 (v) $f = \frac{y}{x-y}.$

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22.1 Find the general solution to the following differential equations:

(i) $\frac{dy}{dx} = \frac{x+y}{x-y}.$
 (ii) $\clubsuit \frac{dy}{dx} = \frac{xy}{x^2-2y^2}.$

22.2 Solve the following initial value problems:

(i) $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}; \quad y(1) = 1.$
 (ii) $\frac{dy}{dx} = \frac{x^3 + 3xy^2}{3x^2y + y^3}; \quad y(1) = -1$

22.3 Find the general solution to the following linear 1st order DEs:

(i) $y' + \frac{y}{x} = x^2 - \frac{1}{x}, \quad y(1) = \frac{1}{4}.$
 (ii) $y' + 2y = e^{-x}.$
 (iii) $\clubsuit y' = x^2 + x^2y$
 (iv) $y' + 3xy = x$
 (v) $y' + \sin(x)y = 3 \sin(2x)$
 (vi) $xy' + y = 2x \sin(x)$
 (vii) $2xyy' = x^2 + 3y^2$
 (viii) $\frac{dy}{dx} + \frac{y}{\tan(x)} = 3x + 1$

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Extra Question! Solve the following DEs:

(i) $(x+y) \frac{dy}{dx} = x + y + 1$
 (ii) $(x+y-1) \frac{dy}{dx} + y - x = 0$
 (iii) $(x-y+1) \frac{dy}{dx} = x - y$

Hint: For (i), try the substitution $v = x + y$. Try similar substitutions for (ii) and (iii).

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OPTIONAL HOMEWORK ASSIGNMENT

Students that wish may submit solutions to five exercises marked with a \clubsuit above. If this improves your continuous assessment mark, it will be included in the final calculation of MA211 grade.

Please submit these *no later than Dec 12th.*