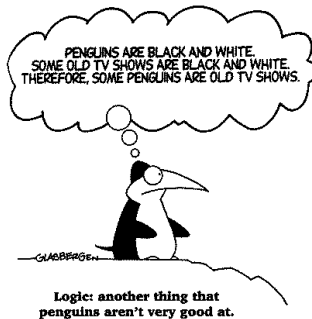


# Mathematical and Logical Aspects of Computing

## Lecture 1: Tuesday, 4th September 2012



# Outline

## 1 Welcome to “Logic”

- Topics
- Text book

## 2 Boolean Operators

- 1-place operators
- 2-place operators
- The conjunction operator
- The disjunction operator
- Non-equivalence

# Welcome to “Logic”

This course is called

*Mathematical And Logical Aspects Of Computing*

But usually, we'll just call it “*Logic*”.

It is (at least) two module codes:

CS304 when taken by Science students

CS310 when taken by Arts students

**Lecturer:** Dr Niall Madden, School of Maths.

Room ADB-1013, Arás de Brún

Email: [Niall.Madden@NUIGalway.ie](mailto:Niall.Madden@NUIGalway.ie), Phone (091 49) 3803.

# Welcome to “Logic”

**Lectures:** Tuesday at **12.10** in the **AM150** and  
Friday at **10.10** in C219 (= ADB 1020). ?

**Tutorials:** Combination of office hours and problem sessions, as well as a revision class before the final exam.

**Web site:** The on-line resources for this course are at  
<http://NUIGalway.BlackBoard.com> and  
<http://www.maths.nuigalway.ie/CS304>. There you'll  
find various pieces of information, including these notes.  
If you are not automatically enrolled onto CS304, please

- Try to enroll yourself. The required access code is:
- Failing that, send me an email, including your ID number.

You need to enroll on blackboard to get announcements, emails, and assessment results.

**Assessment:** Some homework exercises, and written **2 hour** exam at the end of Semester **1**.

The central themes of CS304 include

### 1 Propositional logic:

- How to give a precise mathematical formulation of logical statements;
- How to determine if two statements are equivalent;
- How to establish if two statements are consistent, i.e., don't contradict each other.
- Validity of arguments.
- Boolean algebra.

### 2 Predicate calculus:

- The limitations of propositional logic;
- Quantifiers (existential and universal);
- Semantic entailment; Resolution....

There is no required textbook for CS304, but I will recommend a few for particular topics, including

- Mordechai Ben-Ari, *Mathematical Logic for computer science*. (511.3 BEN)
- John Kelly, *Essence of Logic*. (511.3 KEL)  
Also:
- Stefan Waner and Steven R. Costenoble, *Introduction to Logic*, <http://tinyurl.com/IntroToLogic>
- Ian Chiswell and Wilfred Hodges, *Mathematical Logic* (511.3 CHI).
- Kenneth Rosen, *Discrete Mathematics and its applications* (511.1 ROS).
- Huth and Ryan, *Logic in Computer Science*, 005.1015113 HUT.
- S.N. Burris *Logic of Mathematics and Computer Science*

# Boolean Operators

We are familiar with many different sets, and the basic operators that can be applied to them. [*EXAMPLES*].

For this course we are particularly interested in:

- The set denoted  $\{F, T\}$  (“false” and “true”) and
- the basic operators that we can apply to the two elements of the set above: usually just one or two (later more).



**George Boole** (1815–1864)  
first Professor of Mathematics at UCC.

There are 4 possible operators that take a single argument. They are:

If these, the 1st and 4th are trivial, the 2nd is the *identity operator* and the 3rd is called *negation*, denoted by  $\neg$ , and read as “not”.



There are **16** possible operators that take a two argument. Here are examples of 6 of these:

Of these the most important, other than the identity are

- *Conjunction*, written as  $a \wedge b$ , read as “and”.
- *Disjunction*, written as  $a \vee b$ , read as “or”.

But others are important too – we’ll come back to them later.

The conjunction operator (“and”),  $\wedge$ , is probably the simplest 2-place operator, given that it agrees with its usage of the word “and” in natural language. Here is the table again:

*[EXAMPLE:]* Let's consider the “proposition”

*Today is Tuesday and today we have a Logic lecture*

...(take notes)

The *disjunction operator* (“or”),  $\vee$ , is slightly more subtle, since it does not exactly agree with how we often use the word in natural language.

*[EXAMPLE:]* Let's consider the “proposition”

*Today is Wednesday or today we have a Logic lecture*

...(take notes ... it helps to consider under what conditions would this statement be true, and under what conditions would it be false.)

Finally, consider the proposition:

*I am Arts student or a Science student*

If we were to interpret this in natural language the “truth table” would be:

This is called the “*non-equivalence*”, or, more commonly, the *exclusive or* operator, and denoted as  $\oplus$