

Tuesday, 11<sup>th</sup> Sep 2012

- 1 Recall...
  - Boolean Operators
  - Logical equivalence
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## Recall...

## Logical equivalence

**Assignment:** Giving a particular value of  $F$  or  $T$  to a Boolean variable.

**Interpretation:** a particular assignment to arguments of a Boolean Expression.

**Examples:**

### Logically equivalent

Two Boolean expressions  $E_1, E_2 \in \mathcal{F}$  are *logically equivalent* if  $v(E_1) = v(E_2)$  for all possible interpretations  $v$ . We write this as  $E_1 \equiv E_2$ .

(Informally, we say they are logically equivalent if their truth-table have the same output column.)

Some examples:

- $(a \vee b) \wedge c$  is logically equivalent to  $(a \wedge c) \vee (b \wedge c)$ .
- de Morgan's laws
  - (a)  $\neg(a \wedge b) \equiv \neg a \vee \neg b$
  - (b)  $\neg(a \vee b) \equiv \neg a \wedge \neg b$

## Recall...

## Boolean Operators

### Boolean Operators...

take arguments of *Boolean type* (i.e., with one of the true values  $F$  or  $T$  assigned to them) and maps these to one of  $F$  or  $T$ .

Important operators we've seen so far include:

- **Negation**, written as  $\neg a$ , read as “not  $a$ ” (sometimes :  $\sim a$  or  $\bar{a}$ ).
- **Conjunction**, written as  $a \wedge b$ , read as “and” (sometimes :  $a \& b$ ).
- **Disjunction**, written as  $a \vee b$ , read as “or”.
- **Non-equivalence**, a.k.a., the *exclusive or* operator, and denoted as  $\oplus$  (sometimes:  $\neq$  or XOR)
- **nor**, written as  $a \downarrow b$ ;
- **and**, written as  $a \uparrow b$ ; (sometimes:  $|$  and called the “Scheffer stroke”).

In a few minutes we meet two other *VERY* important ones:

- 1 **equivalence**, written as  $a \leftrightarrow b$ .
- 2 **implication**, written as  $a \rightarrow b$ , and read as “ $a$  implies  $b$ ”;

## Equivalence $\leftrightarrow$

**Warning:** the word “equivalence” is about to be used in a context that is different from “logical equivalence” given above.

The logic table for the “equivalence” operator, written as  $\leftrightarrow$  is:

This operator is sometimes known as “if and only if”, and also is often denoted as  $a \Leftrightarrow b$ .

Can this be written in terms of other operators?

## (Material) Implication $\rightarrow$

Our last example is probably the most important and confusing. It is called “implication” or “material implication”, written as  $a \rightarrow b$ , and read as “ $a$  implies  $b$ ”. The table is:

**IMPORTANT:** there are two ways (that I can think of) to express this operator in “natural language”, and which have can also be determined by checking logic tables:

They are:...

## Venn Diagrams

It has been mentioned that there is a close relationship between the *conjunction* in logic, and *intersection* in set theory. This can be demonstrated using a Venn diagram.

**Example:**

Similarly, there is a relationship between

- *disjunction* and *union*.
- *negation* and *complement*

**Example:**

## Wason's selection task

See Chapter 4 of Chiswell and Hodges “*Mathematical Logic*”

You will be shown 4 cards. Each has a number written on one side and a letter on the other.

You are shown a statement  $S$  about the cards. Then you must answer the following question:

*“Which card or cards must I turn over in order to check whether the statement  $S$  is true?”*

The cards are:

The statement is:

$S$ : *If a card has a vowel one on side, it has an even number on the other.*

## Exercises

1. Sketch the Venn Diagram for all operators mentioned in this lecture.
2. The *reverse equivalence* operator  $\leftarrow$ , has the table

F	F	T
F	T	F
T	F	T
T	T	T

- (a) Show how it can be expressed using  $\leftarrow$ .
  - (b) Sketch the Venn Diagram for it.
3. Show that  $a \downarrow a \equiv \neg a$ ; Show that  $a \uparrow a \equiv \neg a$ ;
  4. Of the following binary operators, which commute and which do not? (Recall, the 2-place operator  $\star$  commutes if  $a \star b \equiv b \star a$ ).
  - (a) exclusive-or  $\oplus$
  - (b) NAND  $\uparrow$
  - (c) material implication  $\Rightarrow$
  - (d) equivalence  $\leftrightarrow$
  5. Show that  $a \rightarrow a \equiv T$  and that  $a \leftrightarrow a \equiv T$ .
  6. Show that  $a \rightarrow b \equiv \neg(a \wedge \neg b)$