Mathematical and Logical Aspects of Computing (CS304/CS310)

http://www.maths.nuigalway.ie/CS304

Lecture 9: Towards Resolution

Tuesday, 2^{nd} Oct 2012

(2/14) Recall: Valid arguments

Recall that an argument:
is **valid** if:

We can use Semantic Tableaux to verify validity as follows:

Examples: (CS304/310 Summer Exam 2011/12)

For each of the following arguments, determine if it is valid or invalid.

(i) If George Boole was Irish, then he was the first Professor of Mathematics at UCC. George Boole was not Irish. Therefore he was not the first Professor of Mathematics at UCC.

 Recall: Valid arguments

(ii) Wittgenstein visited Mayo or Galway. Wittgenstein did not visit both Mayo and Galway. If Wittgenstein had visited Mayo he would have also visited Galway. Therefore Wittgenstein did not visit Mayo.

 Recall: Valid arguments

Here is an example we didn't quite get to last Thursday: *Use the method of Semantic Tableau to determine if the following is a valid argument?*

- Arts or Science students take this Logic course;
- Exactly one of the groups Science and Commerce take logic;
- If Engineering students take this logic course, then Science students do not;
- Engineering students *do* take this logic course;
- We can conclude that Arts students take logic if and only if Commerce students do too.

(5/14) Semantics

What is "semantic" about the Semantic Tableau?

From the Merrian-Webster dictionary

SEMANTICS:

- *the study of meaning...*
- *the meaning, or an interpretation of the meaning, of a word, sign, sentence, etc.: "Let's not argue about semantics".*

From the Oxford Dictionary

Syntax *describes the rules by which words can be combined into sentences, while* **semantics** *describes what they mean.*

(6/14) Towards resolution

The use of logic-table to analyse logical propositions is *semantic*: it deals with (all) possible interpretations of the propositions.

Also, the tableau method is *semantic* since it is an algorithm that finds a particular interpretation that satisfies a proposition (or shows none exists).

Alternatively, we can use a *syntactic* approach.

The classic example is: $\{p \vee q, \neg p \vee q\} \models q$.

Recall: *Disjunctive Normal form*, where we wrote a compound proposition is the **disjunction of conjunctions**.

Now we want to write propositions as the **conjunction of disjunctions**. This is called "*Conjunctive normal form*" (CNF). When a compound proposition in CNF is expressed as a set of propositions with implied conjunction, this is called "*Clause form*".

Example:

Examples:

1. Write $p \leftrightarrow r$ in both **Disjunctive Normal Form** (DNF) and **Conjunctive Normal Form** (CNF).
2. Write $p \oplus r$ in both **Disjunctive Normal Form** (DNF) and **Conjunctive Normal Form** (CNF). ($\longleftarrow$ Exer)

(9/14) Conjunctive normal form

(This slide is based on Ben-Ari, Chapter 4) When we studied DNF in Lecture 4, we did so from a Truth Table. We could use that approach with CNF too. But instead we'll deduce the CNF as follows

1. Rewrite all operators using just AND, OR and NOT.
2. User de Morgan's laws so that any negation applies to atomic propositions, not compound propositions.
3. Eliminate double negations.
4. Use the distributive laws:

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$$

 Conjunctive normal form

Example: Write $(\neg p \rightarrow \neg q) \rightarrow (p \rightarrow q)$ in CNF.

(11/14) Clause form

Recall that with the semantic tableau, we would consider the set of propositions

$$\{A_1, A_2, \dots, A_n\}$$

as a compound proposition

$$A_1 \wedge A_2 \wedge \cdots \wedge A_n$$

If we think of the second line as a single compound proposition, then the first line expresses this proposition is as an *implicit conjunction of proposition*. This is *Clause form*.

(12/14) Resolution

We'll start our study of resolution proper with the following example: *if the following set of propositions consistent as a collection?*

$$\{p \vee q, \neg p \vee s, \neg(q \vee s)\}$$

(13/14) Resolution

The key idea in **resolution** is

If $p \lor q = T$ and $q = F$, then $p = T$.

This can be used to verify that

$$\{p \lor q, \neg q\} \models p.$$

It is useful to view this as a tree:

(14/14) Resolution

Example: Apply resolution to

$$\{\neg p \vee (q \wedge s), q \rightarrow (p \wedge \neg s\} \models \neg p.$$