NUI Galway
OÉ Gaillimh

Summer Examinations 2012/2013

Exam Code(s)	3BA1, 4BCS1, 3BMS2, 3BS9, 4BS3, 4BS4
Exam(s)	Third Arts, Fourth Arts, Third Science, Fourth Science.
Module(s)	Mathematical And Logical Aspects Of Computing
Module Code	CS304, CS310, MA325, MA326
External Examiner(s)	Dr Colin Campbell
Internal Examiner(s)	Prof. Graham Ellis ⋆ Dr Niall Madden
Instructions:	**Answer any 4 questions**.
Duration	**2** Hours
No. of Pages	3 (including this page)
Discipline	Mathematics
Requirements:	N.A.
Release to Library:	Yes

Q1. (a) Outline an algorithm for expressing a proposition in Disjunctive Norm Form (DNF). Explain why this shows that the set $\{\wedge, \vee, \neg\}$ is functionally complete.

Show carefully that the following sets are functionally complete:

(i) $\{\vee, \neg\}$.

(ii) $\{\rightarrow, F\}$.

(b) Construct the truth table for the proposition $\big(a \leftrightarrow b\big) \wedge \big(a \vee c\big)$. Express it in Disjunctive Normal Form (DNF). Show how it can also be represented using a Venn diagram.

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Q2. (a) For each of the following propositions, determine if it is satisfiable, and if it is valid or not valid. If it is satisfiable, give a model.

(i) $a \rightarrow a$.

(ii) $a \rightarrow \neg a$.

(iii) $(a \rightarrow b) \vee (\neg a \rightarrow b)$.

(iv) $(a \vee b) \rightarrow (a \oplus b)$.

(b) Use truth tables to prove or disprove the following identities:

(i) $(a \rightarrow b) \equiv \neg(a \wedge \neg b)$.

(ii) $(a \wedge b) \vee c \equiv (a \vee c) \wedge (b \vee c)$.

(iii) $\big((a \vee b) \rightarrow c\big) \wedge \big(\neg a \rightarrow c\big) \equiv b \rightarrow c$.

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Q3. (a) Use a semantic tableau to establish if the set of propositions

$$\{\neg a \vee b,\ \neg(b \rightarrow c),\ c \rightarrow d,\ \neg(\neg a \vee d)\}$$

is satisfiable as a collection. If it is, then give a model.

(b) What is meant by $\{P_1, P_2\} \models C$?
Explain clearly why, if we want to establish whether or not $\{P_1,\ P_2\} \models C$, it is sufficient to establish whether or not $\{P_1,\ P_2,\ \neg C\}$.

(c) For each of the following, use the tableau method to establish if the following constitute correct logical consequences:

(i) $\{a \rightarrow b,\ b\} \models a$.

(ii) $\{a \rightarrow b,\ \neg b\} \models \neg a$.

(iii) $\{a \rightarrow b,\ b \oplus c\} \models a \rightarrow \neg c$.

Q4. (a) Express the following proposition in Conjunctive Normal Form:

$$\big(\neg a \to \neg b\big) \to (a \to b).$$

Furthermore, show how to write it in Clause Form.

(b) Use the Resolution Procedure to determine if the following set of propositions is consistent:

$$\{a \vee b,\ a \to c,\ b \to c,\ \neg b\}.$$

(c) Show how to express each of the following statements in predicate logic:

(i) Every even number greater than 3 can be written as the sum of two primes.

(ii) There is a number of the form $2^{2^n} + 1$ that is not prime.

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Q5. (a) For each of the following pairs of expressions, determine if A is equivalent to B. Explain your answer.

(i) $A = \forall x(P(x) \to Q(x))$, and $B = \forall x P(x) \to \forall x Q(x)$.

(ii) $A = \forall x P(x) \vee \forall x Q(x)$, and $B = \forall x\big(P(x) \vee Q(x)\big)$.

(iii) $A = \forall x P(x) \wedge \forall x Q(x)$, and $B = \forall x\big(P(x) \wedge Q(x)\big)$.

(b) Use a semantic tableau to show that the following argument is valid:

$$\forall x\big(P(x) \to Q(x)\big) \models \forall x P(x) \to \forall x Q(x).$$

Does the converse hold, i.e., is it true that $\forall x P(x) \to \forall x Q(x) \models \forall x\big(P(x) \to Q(x)\big)$? Explain your answer.

(c) For each of the following arguments, determine if they are valid or not:

(i) All mathematicians are philosophers. Russell was a mathematician. Therefore Russell was a philosopher.

(ii) Everyone works hard or is lucky. If Peter is lucky, then he will pass this course. If James does not work hard, then he will not pass this course. If Peter passes, then James will pass. Therefore James will pass this course.