Summer Examinations 2011/2012

Exam Code(s)	3BA1, 4BA4, 4BCS1, 3BS9, 3CS2, 4BS3.
Exam(s)	Third Arts, Fourth Arts, Third Science, Fourth Science.
Module(s)	Mathematical And Logical Aspects Of Computing
Module Code	CS304, CS310, MA325, MA326
External Examiner(s)	Dr Colin Campbell
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Instructions:	**Attempt 4 questions**.
Duration	3 Hours
No. of Pages	3 (including this page)
Discipline	Mathematics
Requirements:	N.A.
Release to Library:	Yes

Here are some truth tables that may be useful:

Equivalence

a	b	$a \leftrightarrow b$
F	F	T
F	T	F
T	F	F
T	T	T

Exclusive Or

a	b	$a \oplus b$
F	F	F
F	T	T
T	F	T
T	T	F

Nand

a	b	$a \uparrow b$
F	F	T
F	T	T
T	F	T
T	T	F

Nor

a	b	$a \downarrow b$
F	F	T
F	T	F
T	F	F
T	T	F

Q1. (a) Give the truth table for the logical connective $\rightarrow$ (logical implication). Show how it can be expressed using the $\neg$ (not) and $\wedge$ (and) connectives. Give a heuristic justification for this equivalence based on natural language.

(b) State de Morgan's laws, and prove either of them by using a truth table.

(c) Use truth tables to prove or disprove the following identities:

(i) $(a \wedge b) \vee c \equiv (a \vee c) \wedge (b \vee c)$

(ii) $\big((a \oplus b) \oplus b\big) \equiv a$

(iii) $\big((a \leftrightarrow b) \leftrightarrow b\big) \equiv b$

Q2. (a) A university has a rule for a module in logic:

To pass the module you one must have attended at least 18 of the 24 lectures.

Show how to express this rule in propositional logic.

Four students in the class made the following statements:

i. I passed the module;
ii. I attended 20 lectures;
iii. I did not pass the module;
iv. I attended 15 lectures.

Determine which of these must be quizzed further in order to establish that the rule was applied correctly.

(b) For each of the following propositions, find an equivalent proposition in disjunctive normal form (DNF):

(i) $(a \rightarrow b) \wedge (b \rightarrow c)$

(ii) $(a \oplus b) \wedge \neg c$

(c) Explain carefully want is meant by a proposition that is

- *satisfiable*;
- *not satisfiable* (or is a contradiction);
- *valid* (or is a tautology);
- *not valid*.

For each of the following propositions, determine if it is satisfiable or not satisfiable, and if it is valid or not valid.

(i) $a \oplus a$.

(ii) $a \oplus \neg a$.

(iii) $(a \wedge \neg b) \rightarrow (a \vee \neg b)$.

(iv) $(a \uparrow b) \rightarrow (a \downarrow b)$.

Q3. (a) Explain the method of *semantic tableau*. For each of the following collection of propositions, use the tableau method to show that it is consistent, and give a model for the set.

(i) $\{\neg(a \rightarrow b), \quad b \vee c, \quad a \rightarrow c\}$,

(ii) $\{p \rightarrow (q \vee r), \quad q \rightarrow p, \quad \neg r\}$.

(b) Use the tableau method to establish if the following is a valid argument.

$$\{p \rightarrow (q \rightarrow r), \quad (r \wedge s) \rightarrow \neg t\} \models p \rightarrow (s \rightarrow \neg t).$$

(c) For each of the following arguments, determine if it is valid or invalid.

(i) If George Boole was Irish, then he was the first Professor of Mathematics at UCC. George Boole was not Irish. Therefore he was not the first Professor of Mathematics at UCC.

(ii) Wittgenstein visited Mayo or Galway. Wittgenstein did not visit both Mayo and Galway. If Wittgenstein had visited Mayo he would have also visited Galway. Therefore Wittgenstein did not visit Mayo.

Q4. (a) Show how to deduce the following equivalence from de Morgan's law: $\forall x \neg P(x) \equiv \neg \exists x P(x)$. (For simplicity you can assume that the universe of discourse has only three elements.)

(b) Let $S(x, y)$ be the statement "x is a student in the College of y", and let $L(x)$ be the statement "x is a student in this logic course". The universe of discourse for x consists of all the students in NUI Galway, and for y is the colleges of Arts, Commerce, Engineering, and Science.

Express the following in English:

(i) $\forall p\big(S(p, \text{Arts}) \rightarrow L(p)\big)$

(ii) $\forall p \forall q\Big[\Big(L(p) \wedge L(q)\Big) \vee \exists s\Big(C(p, s) \wedge C(q, s)\Big)\Big]$.

Give an alternative representation for the formula in (i) that does not use the universal quantifier ($\forall x$), or the logical implication connective ($\rightarrow$).

(c) Write down the parse tree for the following expression:

$$(x = y) \rightarrow \forall y\Big(\exists z\big(x = F(z, c, w)\big) \wedge P(y, z)\Big).$$

For each occurrence of x, y and z, indicate if it is free or bound.

Q5. (a) In the following expressions, x is free in P and Q, but is not free in y. Determine which of them are correct, and explain your answer.

(i) $\exists x(\neg P(x) \vee y)$ is equivalent to $\neg \forall x P(x) \wedge y$.

(ii) $\exists x(P(x) \rightarrow y)$ is equivalent to $\forall x P(x) \rightarrow y$.

(iii) $\forall x\big(P(x) \vee Q(x)\big)$ is equivalent to $\forall x P(x) \vee \forall x Q(x)$.

(b) Use a semantic tableau to show that $\forall x \forall y P(x, y) \rightarrow P(a, a)$ is valid.

(c) For each of the following arguments, determine if they are valid or not:

(i) All roses smell nice. This flower is a rose. Therefore this flower smells nice.

(ii) All students work hard. Anyone who works hard and is intelligent will pass this exam. I am an intelligent student. Therefore I will pass this exam.