CS304 – CS310
Mathematical and *Logic*al Aspects of Computing

Lecture 6: Logical consequence

Friday, 20^{th} September 2013

Today's topics

1. Decision Procedure
2. Simultaneously satisfiable
3. Logical consequence
4. Exercises

[These slides we revised after the lecture, to remove several of the ones not used in class]

Last week we introduced some new ideas:

We say a proposition A is ***satisfiable*** is there is some interpretation v such that $v(A) = T$. We call v a ***model*** for A.
If a proposition is *not* satisfiable, it is *a contradiction*.

We say a proposition A is ***valid*** if, for all interpretations v, $v(A) = T$.
Equivalently, we say A is "*tautology*". We can write this as: $\models A$.
However, if there is some v such that $v(A) = F$, then we say that A is *not valid* (or "falsifiable"), and we write $\not\models A$

Examples (CS304, Summer 11/12, Q2(c)) For each of the following propositions, determine if it is satisfiable or not satisfiable, and if it is valid or not valid.

(iii) $(a \wedge \neg b) \rightarrow (a \vee \neg b)$

(iv) $(a \uparrow b) \rightarrow (a \downarrow b)$.

Decision procedure

Given an set of formulas, $\mathcal{V}$, an algorithm is a *decision procedure* for $\mathcal{V}$ of given any $A \in \mathcal{F}$, it terminates and reports if $A \in V$ or $A \notin V$.

Probably the most useful example for us is the decision procedure that determines if A is in the set of satisfiable propositions.

- We know such an algorithm exists because for any A because we simply (but tediously) inspect its truth table.
- We can use this procedure to determine if a proposition is *valid*.

Some times it is more efficient to use a *refutation procedure*... (take notes)

Simultaneously satisfiable

A set of formulas $\{A_1, A_2, \ldots, A_n\}$ is *simultaneously satisfiable* if there is some (particular) assignment v such that

$$v(A_1) = v(A_2) = \cdots = v(A_n) = T.$$

We say v is a model for the set.
Equivalently, we say that $\{A_1, \ldots, A_n\}$ is *consistent (as a collection)*.

Examples:

Suppose that

$$\{A_1, A_2, \ldots, A_n, B\}$$

is unsatisfiable. Then...

Definition (Logical Consequence)

Let $\mathcal{V} = \{A_1, A_2, \ldots, A_n\}$ be a set of propositions, and C be a proposition. *C is a logical consequence of* $\{A_1, \ldots A_n\}$ iff every model of $\mathcal{V}$ is a model of C.
We write $\mathcal{V} \models C$.

Examples:

Q1. For each of the following collections of propositions, use a truth table to show that it is consistent, and find an interpretation that satisfies all the propositions in the collection.

(i) $\{a \oplus \neg b, \quad a \vee b, \quad a \uparrow b\}$.

(ii) $\{a \rightarrow \neg b, \quad a \vee b, \quad \neg a \rightarrow b\}$.

(iii) $\{\neg(a \rightarrow b), \quad b \vee c, \quad a \rightarrow c\}$,