CS304 – CS310

Mathematical and *Logic*al Aspects of Computing

Lecture 7: Logical consequences (again)

Tuesday, 24$^{\mathrm{th}}$ September 2013

Today's topics

In Lecture 6 we first met the following concept:

Logical Consequence

Let $\mathcal{V} = \{A_1, A_2, \ldots, A_n\}$ be a set of propositions, and C be a proposition. *C is a logical consequence of* $\{A_1, \ldots A_n\}$ iff every model of $\mathcal{V}$ is a model of C.
We write $\mathcal{V} \models C$.

Example: Use a logic table to show that $\{a \vee b, \neg b\} \models a$.

Example: Use a logic table to show that $\{a \vee b, b\} \not\models a$.

. . . .

Example: Show that $\{a \vee b, a \to c, b \to c\} \models c$.

More examples: What do the expressions $R \models F$ and $\top \models S$ mean? (Here, as usual, $\top$ is "true" and F is "false").

Observation 1.

It is important to note that, if $\mathcal{V} = \{A_1, A_2, \ldots, A_n\}$

$$\mathcal{V} \models C \text{ if and only if } \models \left(A_1 \wedge A_2 \wedge \cdots \wedge A_N\right) \to C.$$

Observation 2.

$\mathcal{V} \models C$ if and only if $\{A_1, A_2, \ldots, A_N, \neg C\}$ is inconsistent.

Suppose the following:

A_1. Ann or Bob have a lecture at 9am on Monday.

A_2. If Bob has a lecture at 9, so too does Cathal.

A_3. Cathal does *not* have a lecture at 9.

Can we conclude that Ann has a lecture at 9?

In general, writing out a full truth table for all formulae in a set is too time-consuming. "*Semantic tableau*" is a faster method to check for consistency.

First recall what a *literal* is:

A **literal** is an atom (i.e., a Boolean variable) or its negation. We call these *positive literal* and *negative literal* respectively.
For a literal p, the pair $\{p, \neg p\}$ is a *complimentary pair of literals*.
For the proposition/formula A, the pair $\{A, \neg A\}$ is a *complimentary pair of formulas*.

Next we use that every 2-place Boolean operator is either

- A **conjunction**, meaning it is true only if *both* atoms have a particular value. There are sometimes called α-formulas.
- A **disjunction**, meaning it is true if *either* atoms have a particular value. There are sometimes called β-formulas.

Examples:

This also applies 2-place operators to (compound) formulas:

- The **conjunction** of two formulas is satisfiable only if **both** are satisfiable.
- A **disjunction** of two formulas is satisfiable if *either* are satisfiable.

Examples:

Q1. For each of the following, use a truth table to establish if it is a correct logical consequence.

(i) $\{a, c\} \models (a \lor b) \lor (b \lor c)$
(ii) $\{q, \quad \neg p \to \neg q\} \models p$
(iii) $\{a \to b, \quad b \to c\} \models a \to c$
(iv) $\{a \to \neg b, \quad b \to \neg c\} \models \neg a \to \neg c$