CS304 – CS310
Mathematical and *Logic*al Aspects of Computing

Lecture 8: Semantic Tableau

Friday, 27^{th} September 2013

Today's topics

1. Recall...
2. The semantic tableau (for propositions)
3. Tableau for collections of propositions
4. Exercises

Over the past two weeks we introduced some terminology:

- A proposition A is *satisfiable* is there is some interpretation v such that $v(A) = T$. We call v a *model* for A.
- If a proposition is *not* satisfiable, it is *a contradiction*.
- A proposition A is *valid* (or is a tautology) if, for all interpretations v, $v(A) = T$. We write this as: $\models A$.
- If there is some v such that $v(A) = F$, then we say that A is *not valid* (or "falsifiable"), and we write $\not\models A$
- A set of formulas $\{A_1, A_2, \ldots, A_n\}$ is *(simultaneously) satisfiable*, or "consistent as a collection" if there is some (particular) assignment v such that $v(A_1) = v(A_2) = \cdots = v(A_n) = T$. We say v is a model for the set.
- Let $\mathcal{V} = \{A_1, A_2, \ldots, A_n\}$ be a set of propositions, and C be a proposition. If C is true for every model of $\mathcal{V}$, then *C is a logical consequence of $\{A_1, \ldots A_n\}$*. We write $\mathcal{V} \models C$.
- $\mathcal{V} \models C$ if and only if $\left\{A_1, A_2, \ldots, A_N, \neg C\right\}$ is inconsistent.

The method we'll study next is mainly used to establish if a *set* of propositions is consistent.
But first we'll look at the simpler case of testing if a (single) proposition is *satisfiable*.

- Write the formula/proposition as the root of a tree;
- Take each in turn,
  - if it is an disjunction (of literals or formulas) expand as 2 branches, one for for each of its arguments.
  - if it is an conjunction (of literals or formulas) expand as a single branch for each of its arguments.

Sometimes called *"derived" rules*:

- If at any stage, a branch contains a complementary pair, we "close" that branch.
- Continue until all branches are closed or all formulas are expanded to literals.
- If any branch remains open, then the proposition is satisfiable, and that branch of the tableau provides a model.
- Otherwise, if all branches are closed, then the proposition is not satisfiable.

Example: For each of the following propositions, write out the tableau and either find a model, or show that it is not valid.

1. $\neg p \wedge p$
2. $\neg q \vee q$

3. $p \wedge (\neg q \vee \neg p)$. ($\longleftarrow$ Source: Ben-Ari, Section 2.6)
4. $a \wedge \neg\Big((b \wedge c) \rightarrow a\Big)$ ($\longleftarrow$ Source: Burris, Exer 2.2.2)
5. $\Big(\ a \vee (b \rightarrow (c \wedge d))\Big) \vee (a \rightarrow b)$. ($\longleftarrow$ Source: Burris, Exer 2.2.2)

Suppose that we wish to check if the collection $\{A_1, A_2\}$ is consistent. That is equivalent to showing that $A_1 \wedge A_2$ is valid. Therefore, we can apply the tableau method to establishing the consistency of collections.

Example: Is the set $\{p \wedge \neg q, p \rightarrow q\}$ consistent?

Example: Is the set $\{a \oplus b, b \to c, a \wedge \neg c\}$ consistent? If so, give a model for the set.

Example: Is the set $\{p \to q, q \to s, \neg p \wedge s\}$ consistent?

Q1. Use a tableau to check for consistency of the following sets of propositions.

(i) $\{a \vee b,\ \neg b,\ \neg a\}$.

(ii) $\{a \vee b,\ b,\ \neg a\}$.

(iii) $\{a \vee b, a \rightarrow c, b \rightarrow c, \neg c\}$.