

CS304 – CS310

Mathematical and Logical Aspects of Computing

Lecture 10: Conjunctive Normal Form

Friday, 4th October 2013

Today's topics

- 1 An example from Lecture 9...
- 2 Semantics
- 3 Towards resolution
- 4 Conjunctive normal form
- 5 Exercises

Here is an example from the end of Lecture 9: **Example: (CS304/310 Exam 2011/12)** (Use the tableau method to) Determine if the following argument is valid:

Wittgenstein visited Mayo or Galway. Wittgenstein did not visit both Mayo and Galway. If Wittgenstein had visited Mayo he would have also visited Galway. Therefore Wittgenstein did not visit Mayo.

What is “semantic” about the Semantic Tableau?

From the Merriam-Webster dictionary

SEMANTICS:

- *the study of meaning...*
- *the meaning, or an interpretation of the meaning, of a word, sign, sentence, etc.: “Let’s not argue about semantics”.*

From the Oxford Dictionary

Syntax *describes the rules by which words can be combined into sentences, while* **semantics** *describes what they mean.*

The use of logic-table to analyse logical propositions is *semantic*: it deals with (all) possible interpretations of the propositions.

Also, the tableau method is *semantic* since it is an algorithm that finds a particular interpretation that satisfies a proposition (or shows none exists).

Alternatively, we can use a *syntactic* approach.

The classic example is: $\{p \vee q, \neg p \vee q\} \models q$.

Recall: *Disjunctive Normal form*, where we wrote a compound proposition is the **disjunction of conjunctions**.

Now we want to write propositions as the **conjunction of disjunctions**.

This is called “*Conjunctive normal form*” (CNF). When a compound proposition in CNF is expressed as a set of propositions with implied conjunction, this is called “*Clause form*”.

Example:

(This slide is based on Ben-Ari, Chapter 4) When we studied Disjunctive Normal Form (DNF) in Lecture 4, we did so from a Truth Table. We could use that approach with Conjunctive Normal Form (CNF) too. But instead we'll deduce the CNF as follows

1. Rewrite all operators using just AND, OR and NOT.
2. Use de Morgan's laws so that any negation applies to atomic propositions, not compound propositions.
3. Eliminate double negations.
4. Use the distributive laws:

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$$

- (a) Write $a \leftrightarrow b$ in both **Disjunctive Normal Form** (DNF) and **Conjunctive Normal Form** (CNF).
- (b) Write $a \oplus b$ in both **Disjunctive Normal Form** (DNF) and **Conjunctive Normal Form** (CNF). (← Exer)
- (c) Write $(\neg a \rightarrow \neg b) \rightarrow (a \rightarrow c)$ in CNF.

Q1. Write the following in Conjunctive Normal Form, and then express in *clause form*

1 $a \uparrow b$

2 $((a \downarrow b) \rightarrow c) \vee (c \rightarrow (a \wedge b))$