10 Class 10: Finite differences in Matlab (1)

(a) Download the Matlab programmes `>> Test_FD.m` and `>> Solve_FD.m` , from the course website, that implement the finite method for solving

$$u''(x) = f(x) \text{ on } (a, b); \quad u(a) = \alpha, u(b) = \beta.$$

Use it to verify that the method is indeed second order.

(b) The `>> for` -loop used in `>> Solve_FD.m` is very slow. Use the `>> sparse` function to do this more efficiently.

(c) Modify the program so that it can solve

$$u''(x) - u(x) = f(x) \text{ on } (a, b); \quad u(a) = \alpha, u(b) = \beta.$$

The method is

$$U_0 = \alpha$$
$$\frac{1}{h^2}\big(U_{k-1} - 2U_k + U_{k+1}\big) - u(x_k) = f(x_k) \quad \text{for } k = 1, \dots, N-1$$
$$U_N + \beta.$$

Is this still second-order accurate?

(d) Next modify the program so that it can solve

$$u''(x) - u'(x) = f(x) \text{ on } (a, b); \quad u(a) = \alpha, u(b) = \beta.$$

For this we need to approximate $u'(x)$. Revise the methods we derived in Lectures 2 and 3, including central differences (D_0), backward differences (D_-), and forward differences (D_+). Which is most accurate?

(e) Finally, modify fy the program so that it can solve

$$\varepsilon u''(x) - u'(x) = f(x) \text{ on } (a, b); \quad u(a) = \alpha, u(b) = \beta.$$

where $\varepsilon = 10^{-2}$. How which method is most accurate?