The Mathematics of Diversity



It is important that an ecosystem be diverse, but what does that mean? Suppose two forests each have the same numbers of trees. One has five different species, but 90% of all individuals are Scots Pine. The other has only four species, but all in roughly equal numbers. Which is more diverse?

Is an area with four different species of tree, all of which are coniferous, more or less diverse than an area with just two species, one coniferous and one deciduous?

There are various ways of answering these questions, usually involving a "Diversity Index". In this project, we'll focus on the concept of Shannon Entropy from information theory. The main tools are mathematical, but a little computation will also be involved.



Algorithms from the Book

Algorithms from the Book, by Kenneth Lange (2020) is freely available through the library at https://search.library.nuigalway.ie/permalink/ f/1pmb9lf/353GAL_ALMA_DS51162627870003626 It features some of the most important algorithms in mathematical and scientific computing, in areas that include graphs/networks, primality, solving linear and nonlinear systems of equations, eigenvalues/vectors, MM methods, data mining, FFT, and Monte-Carlo methods, all with implementations in Julia.

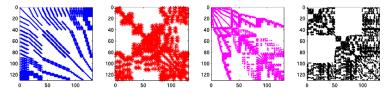
Any of these areas would make for a good starting point for an UG project, with the possibility of emphasising the theoretical foundations, applications, or implementation/programming.

Other ideas

I am interested in supervising any project in the area of **computational mathematics**, and, in particular, algorithms for

(a) solving differential and integral equations

(b) solving linear systems, and/or estimating eigenvalues of large matrices There are numerous opportunities for interesting projects, especially if combined with either **learning** or **mastering** programming, e.g., in Julia, R, MATLAB, Python, C++, etc.



Niall Madden (Niall.Madden@NUIGalway.ie) Final Year and Masters Project Ideas, 2021/2022

Other ideas

Some good starting points

- DeepXDE: A Deep Learning Library for Solving Differential Equations. https://epubs.siam.org/doi/10.1137/19M1274067 A Python package for solving PDEs. Could be applied to financial mathematics.
- Improving the Accuracy of the Trapezoidal Rule. https://epubs.siam.org/doi/10.1137/18M1229353. New insights into classical numerical analysis.
- Rotations in Three Dimensions.

https://epubs.siam.org/doi/10.1137/19M128867X. This 10 page paper would be a good starting point for a project in linear algebra.

Automated solution of Differential Equations by the Finite Element Method ("The FEniCS Book") https://fenicsproject.org/book/