Mathematical and **Logic**al Aspects of Computing (CS304/CS310)

Lecture 20: Semantic Tableaux (for Predicate Logic)

In Lecture 19, we began an investigation of semantic implication for predicate calculus.
Today we'll quickly review the method of Semantic Tableau(x) in propositional logic, so that later we can extend it to predicate calculus.

Recall... consistency

A set of propositions (in propositional logic) is *consistent as a collection* (a.k.a., "simultaneously satisfiable") if there is some model for the set. That means that they can be all true at the same time for some one assignment of $\{T, F\}$ to the atomic propositions.

Examples:

(3/5) Valid arguments (in propositional logic)...

An argument: $\{P_1, P_2, \ldots, P_n\} \models C$, is valid if, whenever the propositions $P_1, \ldots, P_n$ are true, so too is C. (That is, every model for the set $\{P_1, \ldots, P_n\}$ is also a model for C.

Example:

. . . .

As we saw in, say Lecture 6:

The argument $\{P_1, P_2, \ldots, P_n\} \models C$ is valid if the set $\{P_1, P_2, \ldots, P_n, \neg C\}$ is <u>in</u>consistent. (This is a "refutation strategy").

Proof:

(4/5) Semantic Tableau (in propositional logic)

In Semester 1 we learned of a powerful tool ("decision procedure") for finding a model for a set, or showing no such model exists, called *Semantic Tableau*. The idea is to

- Using de Morgan's laws, rewrite proposition so that negation only applies to literals;
- For each proposition, write disjuntions as branching rules, and conjunctions as non-branching rules.
- If a branch has a literal and its negation, it should be closed.
- When finished, if any branch is open it gives a model. If all are closed, the set is inconsistent.

Example: Use the tableau method to show the following set is consistent as a collection.

(5/5) Semantic Tableau (in propositional logic)

Example: Use the tableau method to show the following argument is valid:

. . . .

Example: Use the tableau method to show the following argument is invalid: