CS304/CS310 – Mathematical and Logical Aspects of Computing

Problem Set 3

1. Let $P(x)$ and $Q(x)$ be predicates:

$$P(x) : x \leq 2; \qquad Q(x) : x \geq -2$$

for any $x \in \mathbb{R}$. Determine the truth values of:

(i) $P(1) \wedge Q(1)$.
(ii) $P(0) \rightarrow \neg Q(0)$.
(iii) $(Q(-5) \vee Q(5)) \wedge (P(-5) \vee P(5))$.
(iv) $\exists x \exists y((P(x+y) \wedge P(x-y))$.
(v) $\forall x P(x)$.
(vi) $\exists x \neg P(x) \wedge \exists x \neg Q(\mathbf{x})$.
(vii) $\exists x(\neg P(x) \wedge \neg Q(\mathbf{x}))$.

2. Let $P(x)$ be the statement $x = x^2$. If the universe of discourse is the set of integers, $\mathbb{Z}$, what are the truth values of

(i) $P(-1)$;
(ii) $P(1)$;
(iii) $\forall x P(x)$;
(iv) $\exists x \exists y \Big(P(x) \wedge P(y) \wedge (x \neq y) \Big)$.

3. Suppose the universe of discourse of the propositional function $P(x)$ consists of the integers $\{1, 2, 3, 4\}$. Write out the following propositions without the quantifier symbols $\forall$ and $\exists$:

(i) $\forall x P(x)$; (ii) $\forall x \neg P(x)$; (iii) $\neg \exists x P(x)$.

4. Show how to write the following propositions using only the existential quantifier, disjunctions, conjunctions, and negation:

(i) $\forall x \Big(P(x) \rightarrow \neg Q(x) \Big)$;
(ii) $Q(x) \vee \forall x P(x)$;
(iii) $\forall x \Big(\neg P(x) \leftrightarrow \neg Q(x) \Big)$;

5. Draw a parse tree for each of the following expressions. For each occurrence of x, y and x, indicate if it is free or bound.

a) $\forall x \Big[\exists y \neg Q(x, y) \vee Q(y, x) \Big]$
b) $Q(x, y) \rightarrow \forall y \Big(\exists z((x = z) \vee (x = -z)) \wedge P(y, z) \Big)$.