In this talk, we considered irreducibility testing of nilpotent linear groups over number fields. Let $K$ be a number field and $V \neq 0$ be a finite-dimensional vector space over $K$. Let $G \leq \text{GL}(V)$ be finitely generated and nilpotent. We can assume that $G$ is non-abelian and completely reducible. Our strategy for irreducibility testing of $G$ is built around the following tasks.

(i) Find a proper $K[G]$-submodule of $V$.

(ii) Find a subspace $U < V$ such that $G$ acts irreducibly on $V$ if and only if $\text{Stab}_G(U)$ acts irreducibly on $U$, and $\{Ug : g \in G\}$ is a $G$-system of imprimitivity.

(iii) Find a homogeneous maximal abelian normal subgroup of $G$.

Using congruence homomorphism techniques and Clifford theory, we can always perform one of these tasks. In case (2), we replace $G$ by the induced linear group acting on $U$ and start again. In case (3), we find that the enveloping algebra of $G$ is in an explicit way a crossed product. We can then decide irreducibility of $G$ directly using computational Galois cohomology [3]; this step, however, is usually non-constructive. Hence, we obtain a “partially constructive” algorithm for irreducibility testing of nilpotent linear groups over $K$ [6, §5.5].

In the case of a nilpotent group $G \leq \text{GL}(V)$ which is finite instead of merely finitely generated, we can do much better. In this case, we employ the following variation (based on ideas from [2]) of the above strategy. If $G$ has a non-cyclic abelian normal subgroup, then we can perform task (1) or (2). If, on the other hand, all the abelian normal subgroups of $G$ are cyclic, then the structure of $G$ is sufficiently restricted to allow us to constructively test irreducibility and primitivity of $G$ directly. Consequently, we obtain fully constructive algorithms for both irreducibility and primitivity testing of finite nilpotent linear groups over $K$ [4,5]. Implementations are included in Magma.

References