

## Exam Advice

- Six questions, two hours. Manage the time carefully.
- Same arrangements for MA180, MA190, MA186.
- The three calculus questions will match the three chapters of the lecture notes and the six homework sheets.
- Read the questions carefully. Make sure you answer exactly what is asked, as clearly as you can.
- Think about the learning outcomes when you are preparing.
- Think about communicating to the person who will read your script that you have achieved the learning outcomes.

## Sample Question 2

- (a) State whether each of the following assertions is true or false. Give a short explanation (one or two lines) for your answer in each case.
- Every unbounded subset of  $\mathbb{R}$  is infinite.
  - Every infinite subset of  $\mathbb{R}$  is unbounded.
  - If a subset of  $\mathbb{Q}$  is bounded above, then it has a maximum element.
- (b) State what is means for an infinite set to be *countable*. Show that the set  $\mathbb{Z}$  of integers is countable.
- (c) Let  $S = \left\{ \frac{2n+1}{n^2} : n \in \mathbb{Z}, n \geq 1 \right\}$ .
- List four elements of  $S$ .
  - Show that  $S$  is bounded.
  - Does  $S$  have a minimum element? Does  $S$  have a maximum element? Determine the infimum and supremum of  $S$ .

## Sample Question 1

- (a) Define a function  $A$  by

$$A(x) = \int_1^x \sqrt{t} \, dt$$

for  $x \geq 1$ .

- Draw a diagram that indicates the meaning of  $A(x)$ .
- What does the Fundamental Theorem of Calculus say about the function  $A$ ?
- What is  $A'(4)$ ?

- (b) Evaluate the following integrals.

i.  $\int_1^2 x e^{3x} \, dx$     ii.  $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx$     iii.  $\int \frac{x^2 + 3x + 3}{x + 1} \, dx$

## Sample Question 3

- (a) Give an example of
- a convergent sequence of real numbers;
  - a sequence of real numbers that is bounded and divergent;
  - a sequence of real numbers that is convergent and is not monotonic.
- (b) A sequence  $(a_n)$  of real numbers is defined by

$$a_0 = 4, \quad a_n = \sqrt{a_{n-1}^2 - 2a_{n-1} + 4} \text{ for } n \geq 1.$$

- Write down the first four terms of the sequence.
  - Show that the sequence is bounded below by 2.
  - Show that the sequence is monotonically decreasing.
  - State why it can be deduced that the sequence is convergent, and determine its limit.
- (c) Find the first four terms in the Maclaurin series of  $\frac{1}{1-x}$ .