

Feb 2021 MA180/190/186 - Supplementary Examples on Fundamental Theorem of Calculus

1. (Summer 2018, Q4(c))

Let $A(x) = \int_1^x t \ln t \, dt$, where $x \geq 1$. What is $A'(e)$?

$$\text{FToC: } A'(x) = x \ln x$$

$$A'(e) = e \ln e$$

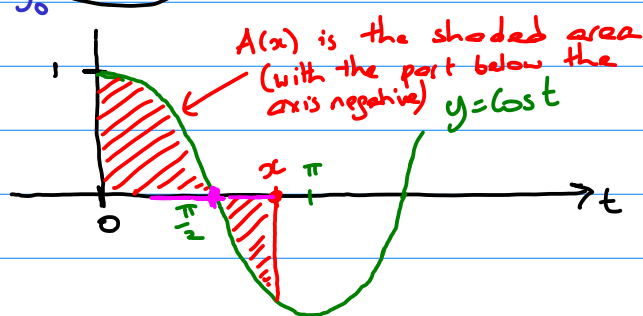
$$= e$$

[Note $\ln e = 1$]

2. (Summer 2015 Q1(a))

Define a function A by $A(x) = \int_0^x \cos t \, dt$ for $x \geq 1$

(i) Draw a diagram that indicates the meaning of $A(x)$



(ii) Show that $A(\pi/2) = 1$

$$A(\pi/2) = \int_0^{\pi/2} \cos t \, dt = \sin t \Big|_0^{\pi/2} = \sin \pi/2 - \sin 0 = 1 - 0 = \boxed{1}$$

(iii) What is $A'(\pi/2)$?

$$A'(x) = \cos x \text{ by FToC} \quad A'(\pi/2) = \cos \pi/2 = 0$$

(iv) What is $A''(\pi/2)$? $A'(x) = \cos x \Rightarrow A''(x) = -\sin x \quad A''(\pi/2) = -\sin(\pi/2) = -1$