Frieze Groups

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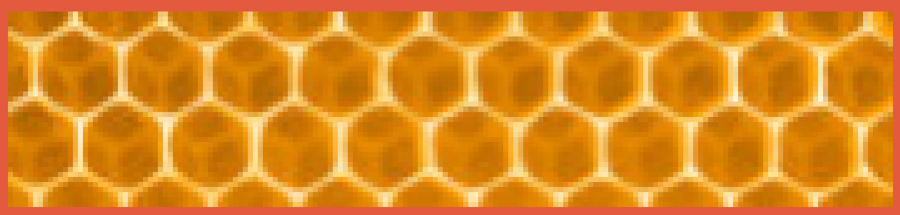
Background and History

Frieze groups describe patterns known as frieze patterns, first observed in artistic architecture and then described mathematically later. Each frieze group contains a few of the symmetries of the shapes as their set of elements and the common group operation is a composition of these elements. Each shape in the frieze pattern can be described as a symmetry of the identity element and the pattern itself is a repeating list of these elements. Frieze patterns were first recognised for their mathematical significance by H.S.M Coxeter in the late 1960's in his book 'Introduction to Geometry (1969)'. The patterns were later recognised as groups by John H. Conway in the late 2000's.
All of these groups are made up of a subset of the set of elements:
Translation - I_d, $\boldsymbol{HorizontalReflection - T_H}$, $\boldsymbol{VerticalReflection - T_V}$, $\boldsymbol{180 - DegreeRotation - R_{180}}$, $\boldsymbol{GlideReflection - G_R}$



Hop (F_1)

'Hop' is the simplest of the Frieze Groups, containing only translation symmetries.
They are commonly seen in architecture and mosaics, but can also be found in art, and even nature as shown in the picture to the left. Set = I_d

Jump (F_2)

'Jump' is another simple group where the repeated pattern is translated repeatedly. Compared to 'Hop', the key difference is that for every appearance of the identity element there is a horizontally reflected copy underneath. A commonly seen example is shown to the right by the leaves of a fern branch with the branch itself acting as the axis of symmetry. Set = I_d, T_H

Step (F_3)

'Step' is the first pattern that we see using glide reflection, which is an element that combines translation and horizontal reflection. Glide reflection causes both of these to be carried out simultaneously to form the elements after the identity.
Set = I_d, G_R

Sidle (F_4)

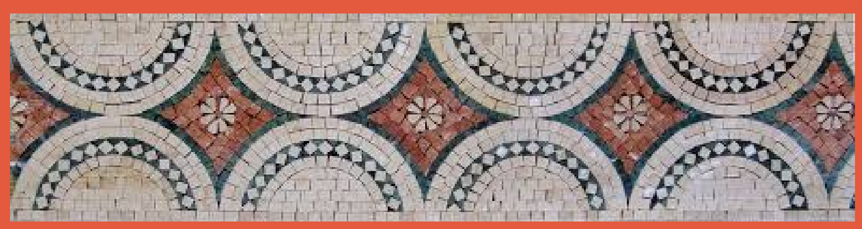
The next frieze pattern is called 'sidle' and consists of repeated vertical reflections of the identity. This pattern is most commonly seen in architecture as shown in this image of a ceiling border.
Set = I_d, T_V

Spinning Hop (F_5)

'Spinning hop' is the pattern which props up in the most interesting places, we found many examples of 'spinning hop' most notably in the attached picture by the artist MC Escher. The pattern can also be seen in the arrangement of phospholipids in the phospholipid bilayer of a cell membrane. Every element is either the identity or a rotation through 180 degrees composed with a horizontal reflection. Set = I_d, $R_{180} o T_H$

Spinning Jump (F_6)

The following mosaic represents 'spinning jump' although how may not be immediately apparent. If you consider the top left corner of the centre shape to be the identity then the whole pattern can be generated using 'spinning hop'. In order the elements are Id,Th,Tv, with Th and Tv alternating after that. Set = I_d, T_H, T_V



Spinning Sidle (F_7)

The last example of a frieze group is 'spinning sidle' it is the least common frieze group, so much so that we couldn't find an example online. Hence why we created our own pattern to demonstrate the properties of this group. In this group the identity element is reflected through the vertical axis and then rotated through 180 degrees. This composed operation is performed repeatedly with 4 iterations returning the pattern to the identity. Set = I_d, T_H, T_V, R_{180}