Frieze Groups

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1. What are the Frieze Groups?

"Frieze groups are two dimensional line groups, having repetition in only one direction."(En.wikipedia.org, 2019). The word frieze comes from architecture which refers to a decorative carving that runs horizontal below a roof line. These patterns extend infinitely so it would be impossible for us to show it all therefore we only show a finite proportion (Mathstat.slu.edu, 2019).

A. Speiser Theorie der Gruppen von endliche Ordung (1927) has the first account of the frieze groups (Burn, 2001).

2. The Seven Frieze Groups

Frieze Groups consist of a combination of operations involving reflections, rotations and one translation. They are discontinuous subgroups of the isometries of the Euclidian plane. There are seven Frieze Groups which we will outline below:

Frieze.JPG Frieze.JPG

1. p1: T (translation only, in the horizontal direction)
2. p1m1: TV (translation and vertical line reflection)
3. p11m: THG (translation, horizontal line reflection, and glide reflection)
4. p11g: TG (translation and glide reflection)
5. p2: TR (translation and 180° rotation)
6. p2mg: TRVG (translation, 180° rotation, vertical line reflection, and glide reflection)
7. p2mm: TRHVG (translation, 180° rotation, horizontal line reflection, vertical line reflection, and glide reflection)

(en.wikipedia.org)

A mathematician called John Horton Conway came up with nicknames for these groups and they are HOP, STEP, SIDLE, SPINNING HOP, SPINNING SIDLE, JUMP and SPINNING JUMP respectively. We will denote each of the Frieze Groups as F1, F2.... F7 for the rest of the poster.

3. Examples of the Seven Frieze Groups

The first frieze group, F1, according to Conway, is also called a HOP. The second frieze group, F2 is called a STEP.The third frieze group, F3 is called a SIDLE. The fourth frieze group, F4 is called a SPINNING HOP. The fifth frieze group, F5 is called a SPINNING SIDLE. The sixth frieze group, F6 is called a JUMP. The seventh frieze group, F7 is called a SPINNING JUMP. (maa.org, 2019)

Groups.JPG Groups.JPG

4. Frieze Groups throughout History

The human brain is hardwired to look for symmetry, and consider it beautiful. Throughout the whole history of art, symmetries have been used in many forms, and one of them are the frieze patterns. Artist have been using complex patterns without realising the mathematics behind it for millennia. Frieze patterns have been observed as far down the history, as cave paintings. Mathematics behind the cave paintings might be debatable, but if we fast forward towards ancient Greek and Persian times, more complex patterns start appearing, mainly in architecture and fabric design. Frieze patterns have been considered aesthetically very valuable. They influenced and inspired many artists and designer in many different cultures and times, and remained an inspiration even in modern days.

Real-Life.JPG Real-Life.JPG

Swimming Stags – Lascaux (cca 15000b.c.)

Achaemenid frieze designs at Persepolis

A repeating frieze pattern designed with rocks on a path leading to the Jewish synagogue on the island of Rhodes

Pazyryk Carpet, 5th century BCE

Celtic wood burnt box

Lakota (Sioux) blanket band circa 1870

Sarthak Estate Developers design

Modern day clothes

6. References

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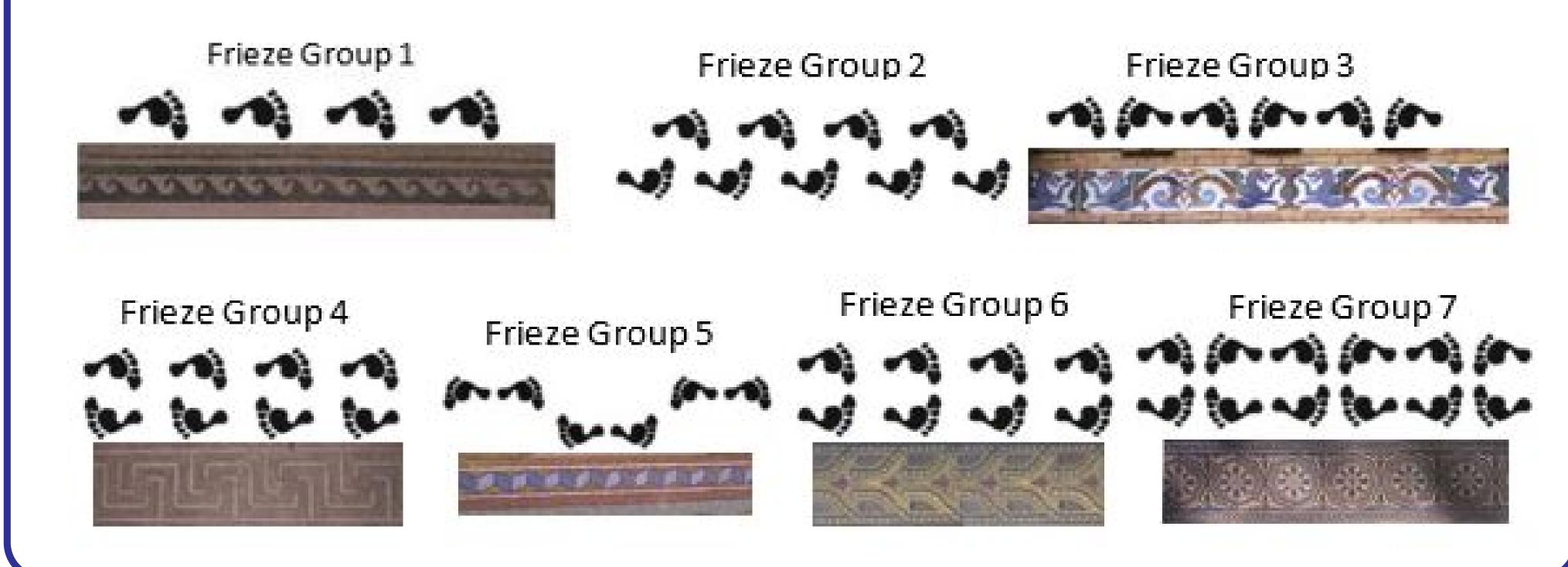
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5. Conclusions

After carrying out this poster we learned a lot about the frieze groups

1. There are only 7 types of frieze patterns.
2. A frieze group is a class of infinite discrete symmetry groups of patterns on a strip.
3. The frieze groups are very common in real life more than we would expect. Some examples are types of a car, frame of a mirror, designs on buildings.
4. Frieze patterns occur in geometry when we are dealing with the four types of transformations. All frieze groups are geometric patterns.

Frieze Groups Video   Frieze Groups Webpage

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