The Group of Symmetries of A Cube

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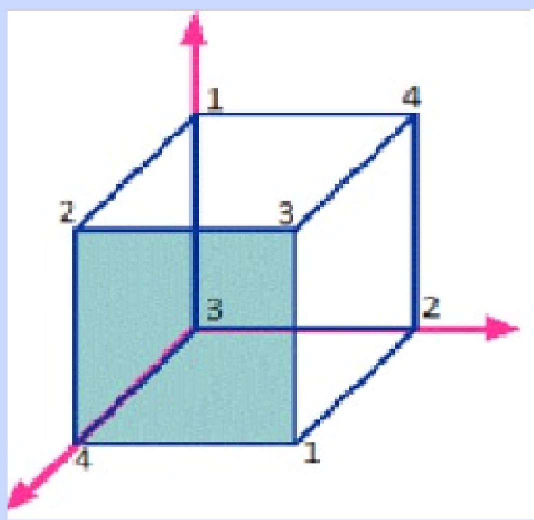
Overview

The group of symmetries of a cube is isomorphic to S4,the group of permutations of four objects. This means that if we number the vertices of the cube from 1 to 4, and where opposite vertices are given the same number, the permutation corresponding to a symmetry can be read out from one of the faces. There are 48 symmetries in total. 24 of these are rotational symmetries. The other 24 symmetries come from the reflections of the cube that are isometries but which can not be carried out physically in a 3-dimensional space. Rotations in 3D are non-commutative because rotation changes direction of every potential other axis except itself, hence rotational symmetry in 3D is non-abelian.



Overview2.jpg Overview2.jpg

What is the Group S_4?

The symmetric group S_4 is the group of all permutations of 4 elements(Wikiversity, 2019). It has 4!=24 elements and is non-abelian. S_4 contains 2-cycle permutations, product of 2-cycle permutations, 3 and 4-cycle permutations.

References

[1] GeoGebra. (2019). Rotational Symmetry of Cube. [online] Available at: https://www.geogebra.org/m/rBN6v4TH [Accessed 21 Nov. 2019].

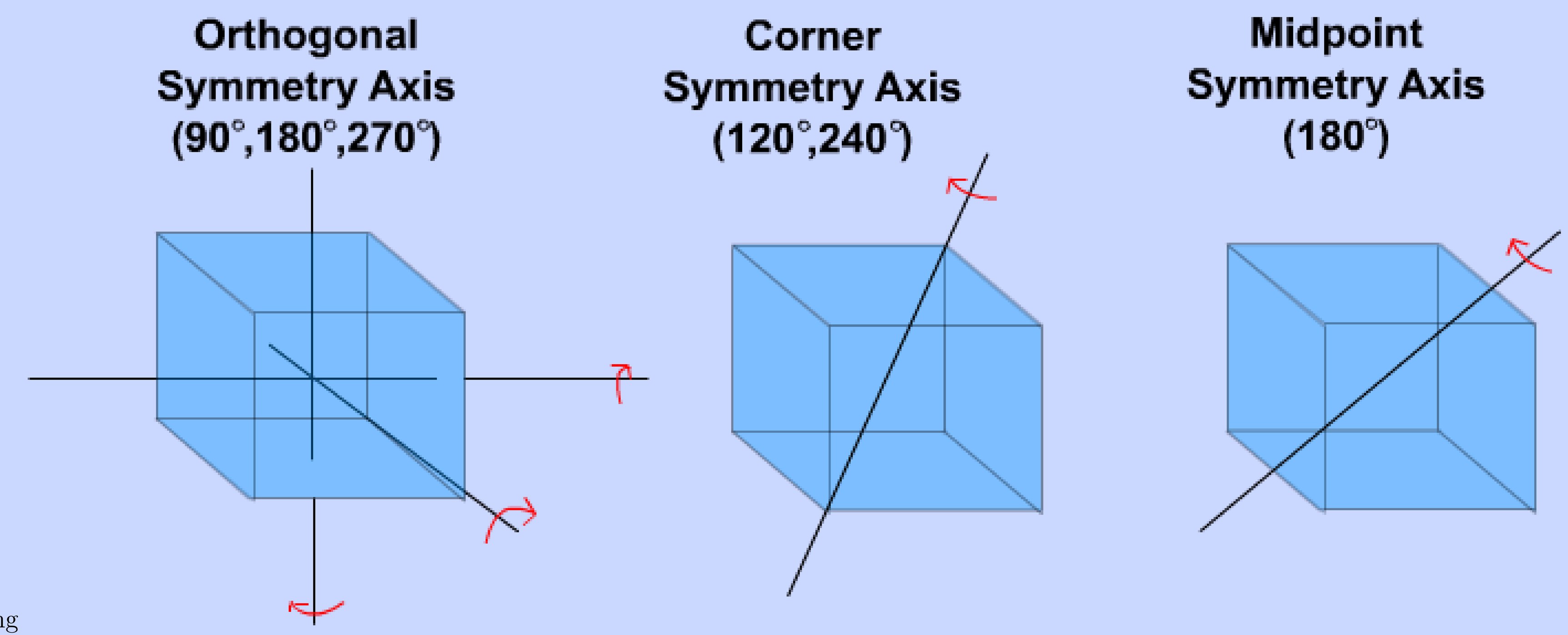
[2] Wikiversity. (2019). Symmetric group S4 - Wikiversity. [online] Available at: https://en.wikiversity.org/wiki/SymmetricgroupS4 [Accessed 21 Nov. 2019].

[3] Anon, (2019). [ebook] University of Oslo. Available at: https://www.uio.no/studier/emner/matnat/math/MAT2200/v15/oppgave2.pdf [Accessed 21 Nov. 2019].

[4] Math.brown.edu. (2019). Duals of Regular Polyhedra. [online] Available at: http://www.math.brown.edu/banchoff/Beyond3d/chapter5/section03.html [Accessed 23 Nov. 2019].

Rotational Symmetries of a Cube

There are 24 rotational symmetries of a cube. These include the identity rotation, 9 combinations of rotations about the central axes, 8 combinations of rotations about the diagonal vertices and 6 rotations about the diagonal midpoints. To find these, we used GeoGebra (GeoGebra, 2019) and created our own model.



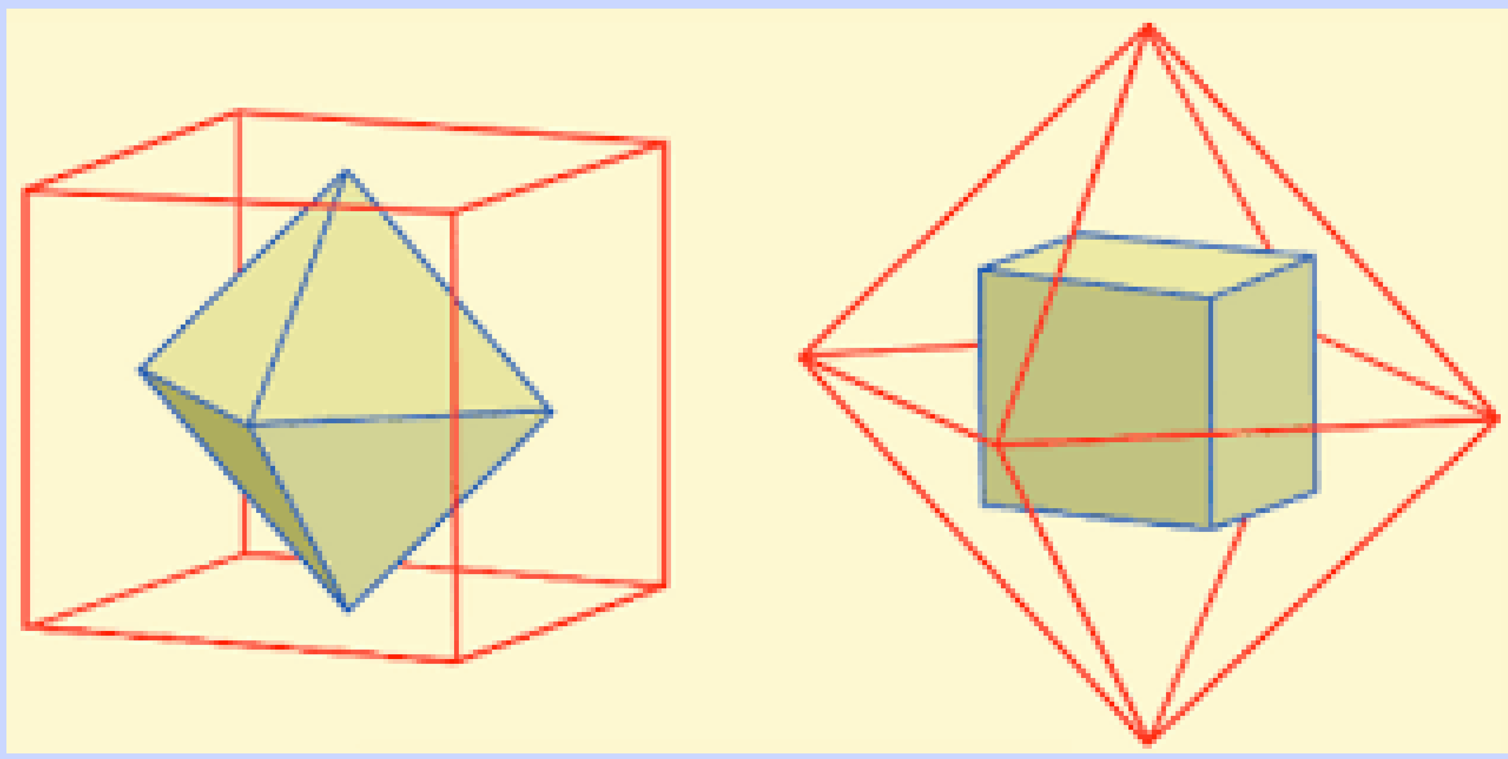
Rotational Symmetry.png Rotational Symmetry.png

The symmetries can be permutated in the following way when labelled like the image in panel 1:

Centre:(1234), (13)(24) (1432); (1324), (12)(34), (1423); (1243), (14)(23), (1342)
Diagonals:(1)(243), (1)(234); (2)(143), (2)(134); (3)(142), (3)(124); (4)(132), (4)(123)
Midpoints:(14), (12), (23), (34), (24), (13)

Symmetries of a Cube and Octahedron

The cube and the octahedron both have 48 symmetries, divided into 24 rotational symmetries and 24 using a rotation and a reflection. This can be explained by the duality of the two shapes. Essentially, the midpoint of the 6 faces of the cube corresponds to the 6 vertices of the octahedron, and the 8 midpoints of the faces of the octahedron correspond to the 8 vertices of a cube.



The 24 rotational symmetries of the octahedron can then be explained in terms of the cube. It has 9 rotations of 90 degrees through its vertices the same way the cube has nine through the centre of its faces. There are 8 rotations of 120 degrees through the centre of the faces, similar to the 8 through the diagonals of the cube and finally 6 through the midpoints of the edges, identical to the cube.