MA3343 Project on Wallpaper Groups

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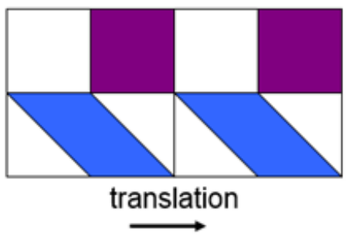
What is a Wallpaper Group?

Wallpaper groups, also known as plane groups, describe the possible symmetry compositions of two-dimensional repetitive patterns in the plane. These symmetries transform the patterns on the plane while preserving distance. This type of symmetry is known as an isometry, of which there a four types: Translations, Rotations, Reflections and Glide Reflections.

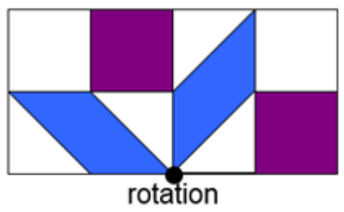
Translation

Translations shift the image to another point in a plane without changing its orientation.



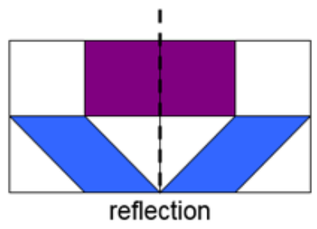
Rotation

Rotations turn the image some angle around a fixed point in the plane.



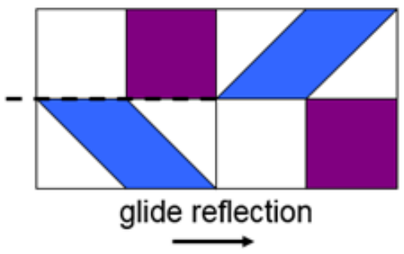
Reflection

Reflections create a mirror image by flipping the image across an axis in the plane.



Glide Reflection

Glide reflections are reflections and translations combined.



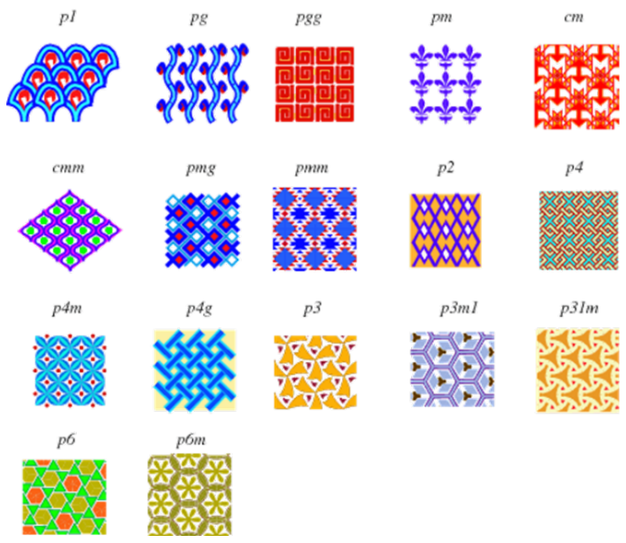
Some Real Life Examples of these Symmetries

A common example where one can see a glide pattern is found looking overhead at rowers in a boat. Certain animals even display these patterns on their skin, for example Kaiser's spotted newt. We can see a glide reflection in the white pattern along its back which then into a reflection as it reaches its tail.

Images

17 Groups

There are 17 different types patterns that can be formed using different combinations of the symmetries. These patterns make up the 17 different wallpaper groups pictured below. Evgraf Fedorov proved that there can only be 17 distinct wallpaper groups, this was proved independently in 1924 by George Pólya.



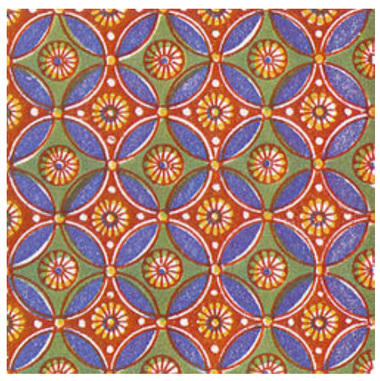
Notation

Each group is referred to using the following notational structure: There are four symbols used when each group is described in full. The first symbol is always either a p or a c, referring to that group's lattice type – either primitive or centred. This is followed by three symbols which define the group's symmetry elements. The first of these three signifies the highest order of rotational symmetry. The second and third denote the symmetry elements in the x and y axes of the plane, either m for mirror or g for glide. The number 1 is used is there if no symmetry element present in either direction. Sometimes not all three symbols are used to differentiate between groups, as they are not necessary to include. For example, if there is no rotational symmetry present, the first symbol is not needed. E.g. p2mm = pmm.

References

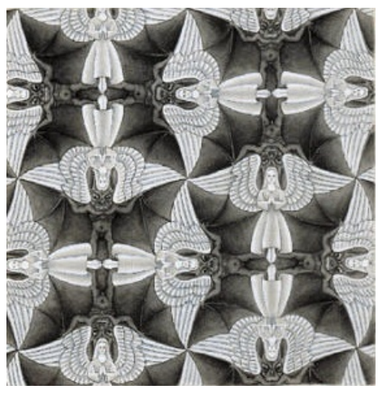
1. https://mathstat.slu.edu/escher/index.php/Wallpaper_Patterns
2. https://en.wikipedia.org/wiki/Wallpaper_group
3. https://www.youtube.com/watch?v=5UbMFiK3LY0
4. https://www2.clarku.edu/faculty/djoyce/wallpaper/trans.html

Egyptian



This Egyptian design is described by the group p4m. It contains reflections and 4-fold rotation centers lie on reflection axes.

Escher Sketch



Escher's Sketch 45- largest rotation order 4, also contains rotation order 2; since the only reflection axes are horizontal and vertical, the symmetry group of this pattern is p4g

Christmas Paper

You may even see the patterns of a wallpaper group when wrapping your gifts for Christmas this year! This pattern represents the group pg.