

Wallpaper Groups

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1. Introduction

A wallpaper group is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. They are found in architecture and decorative art, especially in textiles and tiles as well as wallpaper. A proof that there were only 17 distinct groups of possible patterns was first carried out by Evgraf Fedorov in 1891[1] and then derived independently by George Pólya in 1924. Wallpaper groups categorize patterns by their symmetries. Subtle differences may place similar patterns in different groups, while patterns that are very different in style, color, scale or orientation may belong to the same group.

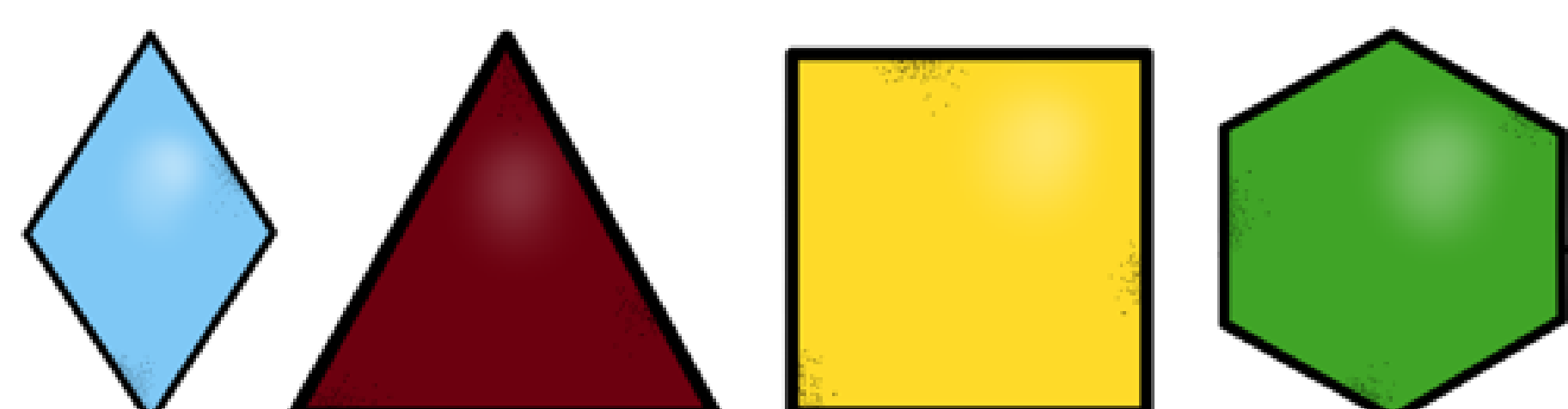
2. Rotations

Rotations have different orders of either 2 (180 degrees), 3 (120 degrees), 4 (90 degrees), 6 (60 degrees).

Oftentimes there may be a combination of reflections with different orders or reflections.

Wallpaper groups can use both the positive and negative of a rotation.

Real life examples: The wheels on a bus go around and around, the earth's orbit around the sun

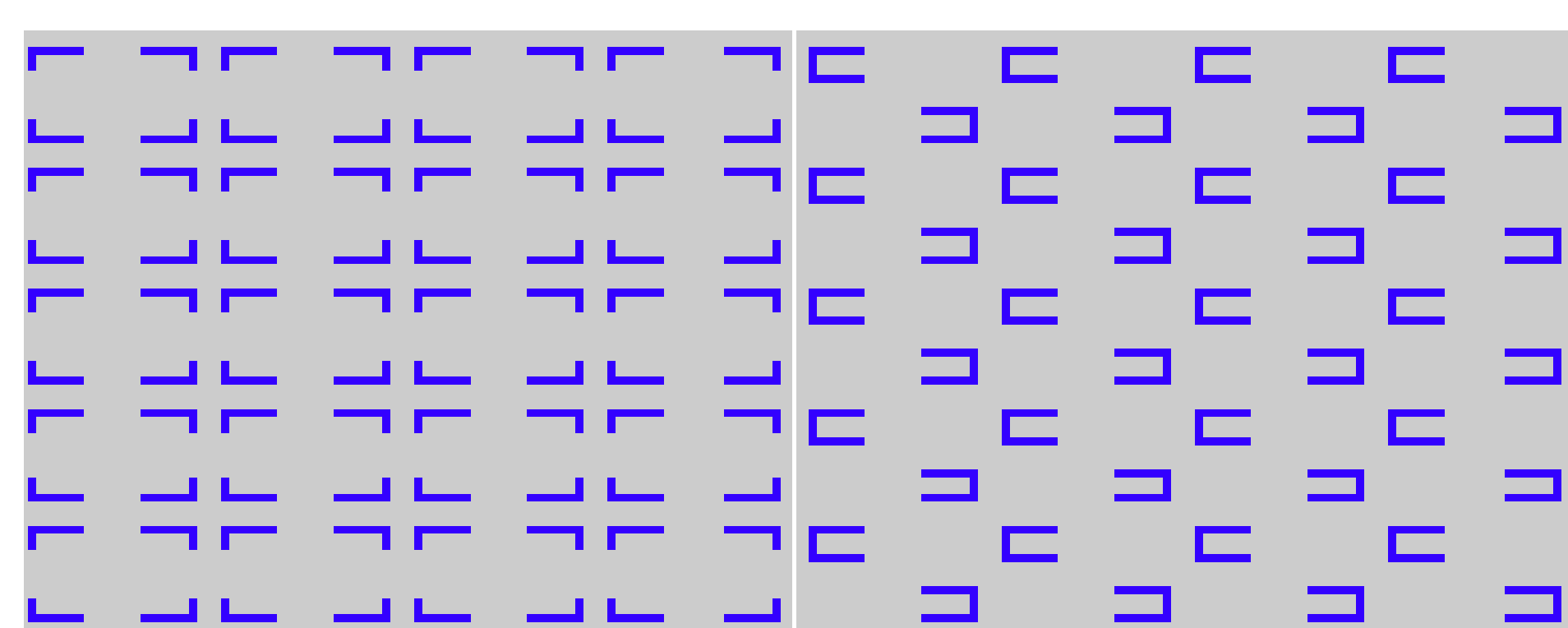


3. Reflections

There are two types of Reflections

Axis of reflection: examples butterfly, aeroplane

Axis of Glide reflections : consists of a reflection and THEN a translation along the line, combined into a single operation. Real life examples are footsteps, Conways game of life



7. References

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4. Examples of Wallpaper

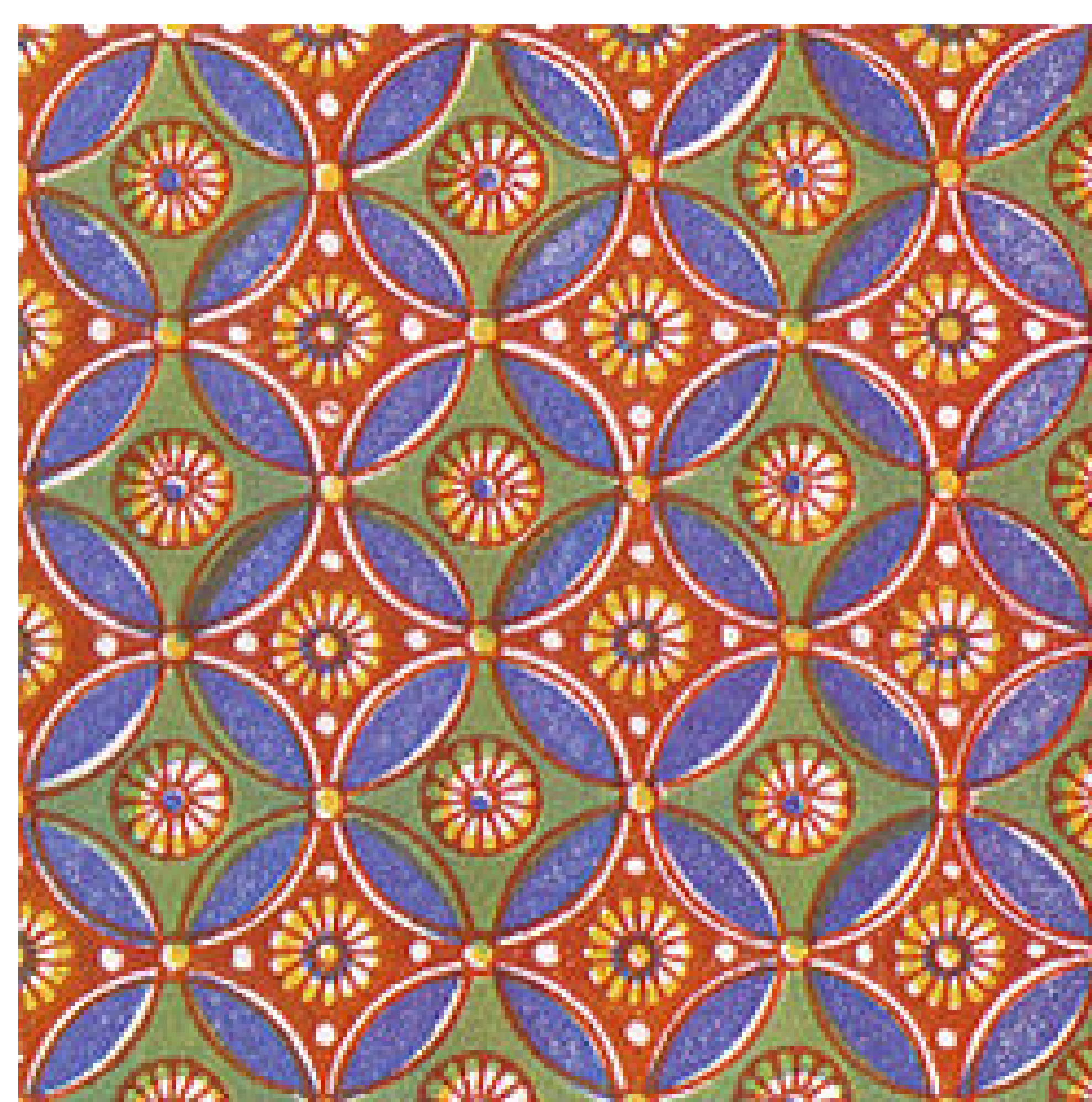
Egyptian Design This is an example of a wallpaper group of $pm4$

There are four reflections horizontal, vertical and diagonal.

The group has two rotation centres of order four 90.

Rotations of order two 180 are centred at the intersection of the glide reflection axes.

Every Rotation centres lie on the reflection axes

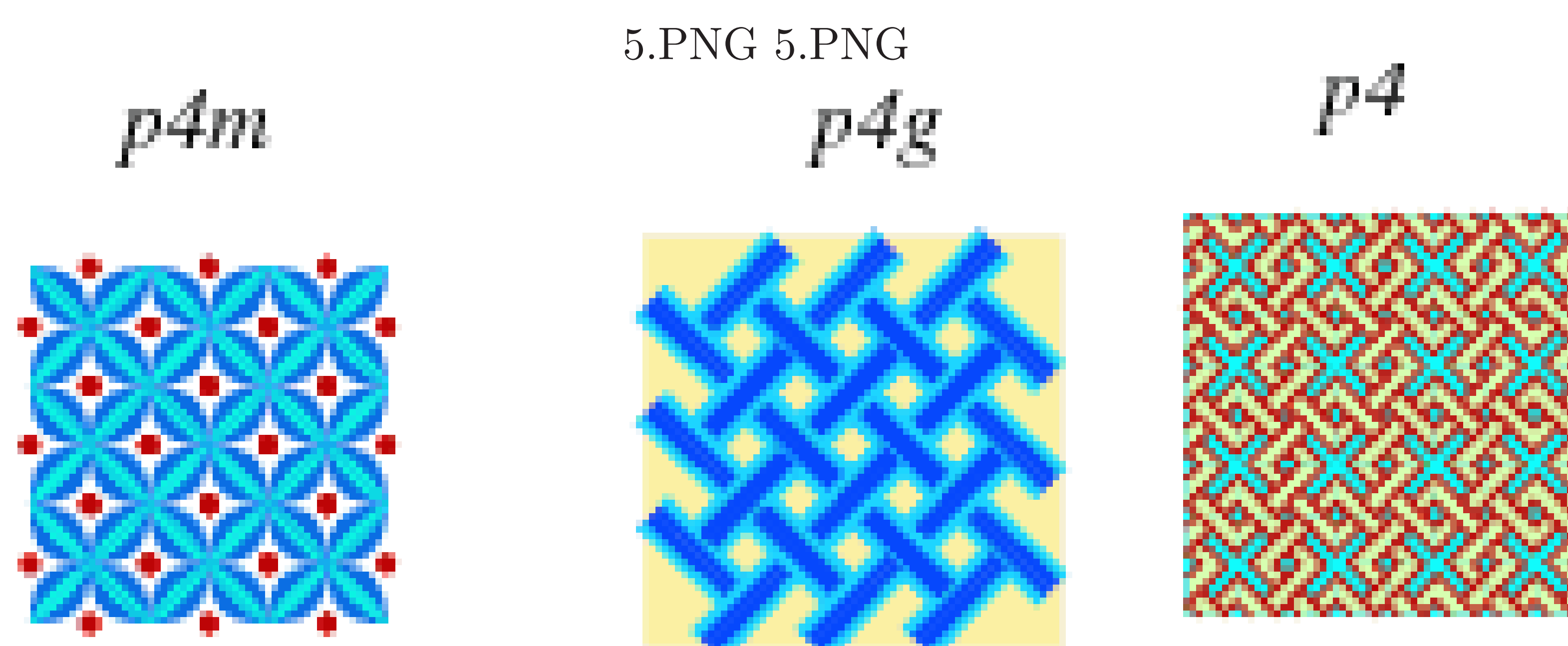


(1).jpg (1).jpg

5. Comparing Wallpaper Groups

The wallpaper groups $p4$, $p4m$ and $p4g$ have square lattice paths and they are the groups that contain 90 degree rotation. $P4m$ and $P4g$ have reflections as well as the 90 degree rotation unlike $P4$ which has no reflections.

This group can be spotted visually by relating lines of reflection to centers of rotation. $P4m$ is generated by the translation subgroup and the point group, therefore this group contains reflections parallel to the sides of a fundamental domain which is a square. For $P4m$ all the centers of rotation lie on reflection lines. The group $P4g$ contains reflections in 2 perpendicular directions (180 and 90 degrees). For $P4g$ not all centers of rotations lie on reflection lines.



6. Conclusions

In conclusion, wallpaper designs have been around for a long time, wallpaper was first used in China in 200B.C. As the Chinese passed this knowledge to other cultures, the uses and methods of making paper improved with each generation and as other cultures added their own touches.

From then the design of wallpaper has improved and after WWII, wallpaper was improved with plastic resins which made it more durable, washable and stain resistant.