The Mathematics behind Frieze Groups

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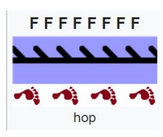
What are Frieze Groups?

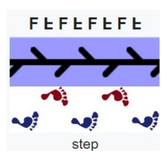
Frieze groups bring the relationship between mathematics and art to life. A frieze is a design on a two-dimensional surface that is repetitive in one direction. A frieze group is the set of symmetries of a frieze pattern. Frieze patterns have been popular in art and architecture throughout history - from the far east to the west, the beauty of frieze patterns has stood the test of time. The mathematics behind the artistic splendour, while less obvious is even more impressive. Mathematically speaking a frieze group is a class of infinite discrete symmetry groups of patterns on a band. Studying frieze patterns reveals that they can be classified into seven types according to their symmetries. This poster describes the characteristics of each of the seven groups and presents some examples from art and architecture

The seven frieze groups

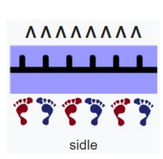
Translations only (p1): This group is only generated by a translation by the smallest distance over which the pattern is periodic.



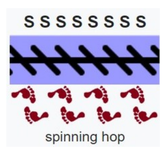
Glide-Reflections and Translations (p11g): This group is only generated by a glide reflection, with translations being obtained by combining two glide reflections.



Vertical Reflection lines and Translations (p1m1): This group is generated by a translation and a reflection in the vertical axis.



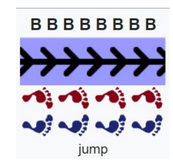
Translations and 180 degree Rotations (p2): This group is generated by a translation and a 180 degree rotation.



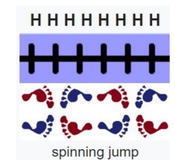
The seven frieze groups

Vertical Reflection lines, Glide reflections, Translations and 180 Rotations (p2mg): This group is generated by a glide reflection and either a rotation or a vertical reflection.

Translations, Horizontal Reflections and Glide Reflections (p11m): This group is generated by a translation and the reflection in the horizontal axis. The glide reflection here arises as the composition of translation and horizontal reflection



Horizontal and Vertical Reflection lines, Translations and 180 Rotations (p2mm): This group requires three generators, with one generating set consisting of a translation, the reflection in the horizontal axis and a reflection across a vertical axis.



Examples

The first use of frieze patterns was in ancient Mesopotamia (modern day Syria and Iraq) where cylinder seals were used to imprint a pattern onto a wet clay tablet. Thanks to their beauty and popularity one never has to look too far to find frieze patterns. The finest Irish example of a frieze pattern in architecture is the GPO. On the façade of the building there is an ornate frieze that is translations (p1) only. Frieze patterns develop naturally like in honeycomb where the hexagonal structure is p2mm. One of the last great Greek mathematicians, Pappus of Alexandria that "bees possessed a divine sense of symmetry."

Cylinder Seal

GPO

Honeycomb

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NUI Galway

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